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A SEGMENT DESIGN METHOD FOR REINFORCED CONCRETE SLABS

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WILLIAM J. KERR

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled A SEGMENT DESIGN METHOD FOR REINFORCED CONCRETE SLABS submitted by WILLIAM J. KERR in partial fulfilment of the requirements for the degree of MASTER OF SCIENCE in CIVIL ENGINEERING.

Abstract

This thesis presents a lower bound segment method for the design of reinforced concrete flat slabs. The investigation focused on the development of the methodology and recommendations for the use of the procedure to aid the designer in producing safe and serviceable slab designs. A brief look at previous lower bound design methods for slabs is included and the results summarized.

The procedure consists of dividing the slab into rectangular segments bounded by lines of zero shear. For various support conditions boundary moments are applied to satisfy equilibrium and moment compatibility between segments. For each segment type, solutions obtained from an elastic finite element analysis were used to evaluate the internal distribution of moments corresponding to different boundary moment configurations. Three design examples are presented to introduce the method and to show its applicability to regular and irregular slab problems. The resulting moment fields are compared to elastic solutions.

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List of Symbols

As	Area of steel per unit width of slab.
b	Width of critical slab section.
c	Diameter of circular column.
c_1	Side length of rectangular column in the direction of span being considered.
c_2	Side length of rectangular column perpendicular to the direction of span being considered.
d	Effective depth of slab, from slab surface to centroid of tensile reinforcement.
$f'c$	Specified strength of concrete.
fy	Specified yield strength of reinforcement.
h	Slab thickness.
k	Ratio of exterior column to interior column stiffnesses in an exterior slab panel.
ℓ	Center to center span.
ℓn	Clear span in the direction under

consideration.

m_x	Moment in x-direction, (reinforcement parallel to x-axis), per unit width of slab.
m^+_x	Positive moment in the x-direction per unit width of slab.
m^-_x	Negative moment in the x-direction per unit width of slab.
m_y	Moment in y-direction, (reinforcement parallel to y-axis), per unit width of slab.
m^+_y	Positive moment in the y-direction per unit width of slab.
m^-_y	Negative moment in the y-direction per unit width of slab.
m_{xy}	Twisting moment per unit width of slab.
M'	Total static moment acting on a panel.
M^o	Total static moment acting on a slab segment.
M^+	Positive portion of M^o.
M^-	Negative portion of M^o.
M^o_x	Total static moment acting on a slab segment in the x-direction.

M^o_y	Total static moment acting on a slab segment in the y-direction.
M^+_x	Positive portion of M^o_x .
M^-_x	Negative portion of M^o_x .
M^+_y	Positive portion of M^o_y .
M^-_y	Negative portion of M^o_y .
Mu	Ultimate moment capacity at a section.
M_α	The value of M^o_y for that portion of the edge within αs_x of the support.
$M_{1-\alpha}$	The value of M^o_y for that portion of the edge beyond αs_x of the support.
M_ε	The value of M^o_x for that portion of the edge within ϵs_y of the support.
$M_{1-\varepsilon}$	The value of M^o_x for that portion of the edge beyond ϵs_y of the support.
q	Uniformly distributed load applied to slab.
q_x	Portion of q carried by slab in the x-direction.
q_y	Portion of q carried by slab in the y-direction.

Ql	Lower bound load.
Qu	Upper bound load.
s	Side length of rectangular segment.
s_x	Side length, parallel to x-axis, of rectangular segment.
s_y	Side length, parallel to y-axis, of rectangular segment.
W	Total load acting on a panel.
α	Portion of s_x over which the negative edge moments act on a segment.
ϵ	Portion of s_y over which the negative edge moments act on a segment.
ρ	Tension reinforcement ratio (As/bd).
ϕ	Capacity reduction factor.
ψ	Ratio of negative to positive edge moments acting on a segment.

1. Introduction

1.1 Purpose and Scope

The Segment Design Method is a lower bound procedure which is presented as a hand calculation method suitable for routine design office use, for designing two-way, reinforced concrete slabs subjected to uniform load. The method involves dividing the slab into a number of suitable rectangular segments, each being independently supported, either by a wall or column. For each segment a distribution of edge moments is found which satisfies equilibrium requirements. The designer then assembles the segments ensuring compatability of moments at the boundaries of adjacent elements by adjusting, if necessary, the equilibrium edge moments. Reinforcing steel is selected to resist these moments to complete the design.

The methodology is not intended to give an exact or unique solution but to provide a moment field which satisfies equilibrium and results in a reinforcement pattern that provides adequate serviceability. The main advantages to a design method of this type are firstly, the design moment field is found solely from equilibrium, resulting in very simple moment expressions and secondly, it is a lower bound procedure and so the design is always on the safe side, strength-wise.

The thesis itself is subdivided into four main parts. The first part is in the form of an introduction to lower bound methods for slab design. Both the upper and lower bound theorems are given for comparative purposes. A short history of the development of lower bound methods is presented starting with the work by Nichols and the use of this in developing the Direct Design Method. This is followed by a brief explanation of Hillerborg's strip method and finally a short discussion on the segment methods developed by Hillerborg and Wiesinger.

The second section of the paper deals with the design methodology. The procedure for carrying out the design is presented, first in point form and then followed by a detailed explanation of each step complete with guidelines and recommendations for the designer. The method is streamlined for simplicity and the design equations are straightforward and easy to apply.

The third part deals with the individual segments, their properties and characteristics. The selection of these segments or elements was influenced by the types of problems a designer would generally expect to encounter on a regular basis. To handle the case of flat slabs supported on columns a corner supported segment was developed. This type of segment is rectangular in shape and supported at one corner only. For slabs supported on walls two segments were developed, one an edge supported segment and the other an adjacent edge supported segment. A detailed discussion of

the three segments is included in chapter 3.

Finally, the last section, chapters 4 and 5, include design examples which illustrate the method of design and the use of the recommendations and guidelines given previously. Chapter 4 deals with the design of flat plate structures supported only on columns and chapter 5 includes a design example involving both column and wall supports.

1.2 Historical Background

1.2.1 Plasticity Theorems

All plastic methods of design or analysis fall into one of two catagories, either based on a lower bound or upper bound theorem. These two theorems, as they apply to reinforced concrete slabs, have been stated by Hillerborg(1) and are given below.

> *Lower-bound theorem*: If there is a load *Q1* for which it is possible to find a moment field which fulfills all equilibrium conditions and the moment at no point is higher than the yield moment, then *Q1* is a lower-bound value of the carrying capacity. The slab can certainly carry the load *Q1*.

> *Upper-bound theorem*: If, for a small virtual increment of deformation, the internal energy

absorbed by the slab on the assumption that the moment at every point where the curvature is changed equals the yield moment and this energy is found to equal the work performed by the load *Qu* for the same increment of deformation, then *Qu* is an upper-bound value of the carrying capacity. Loads greater than *Qu* are certainly high enough to cause moment failure of the slab.

A lower bound type procedure underestimates the collapse load, thus being always on the safe side, whereas an upper bound procedure overestimates the collapse load. An essential requirement for having a lower bound solution is that at no point in the slab are the yield moments exceeded. This requires that the moment field at all points be evaluated to ensure that such is the case. It also provides enough information to detail all reinforcement requirements while an upper bound solution only identifies moments at discrete locations.

1.2.2 Development of Lower Bound Design Methods

Through the early 1900's the design of flat slabs was based on load tests and past performance. In 1914, Nichols(2) attempted to rationalize the basis behind the selection of moments for slabs supported on columns by showing that the total moment in a panel could be determined

from the equations of equilibrium. He considered a typical interior panel of a slab consisting of an infinite array of equal square panels supported on circular columns.

By defining a segment bounded by the panel centerlines, the column centerlines and the column periphery Nichols derived an expression for the total moment about a column centerline acting on that segment using the assumption that the total shear at the column face was uniformly distributed. A simplified version of his formula is given below, where M′ represents the total moment in a panel and W the total load. The other variables are as defined by Fig. 1.1.

$$M' = 0.125W\ell(1-2c/3\ell) \tag{1.1}$$

Nichols′ attempt at establishing the total moments on a panel was not well received. There were several designers at the time who had built slabs with less total moment and who could argue that their structures were quite adequate. As a result of these arguements when an expression for the panel moment first appeared in the ACI Building Code in 1921 the format was the same as that proposed by Nichols but the coefficient was reduced to 0.09. This resulted in a significantly lower static design moment which persisted for half a century. Since 1971 the code expression reflects more closely Nichol′s original equilibrium equation.

It is to be noted that although Nichols conceived of the idea of dividing a panel section into segments and obtaining the total required panel moment, he did not indicate the distribution of this moment, either along the segment boundaries or inside the segment. Therefore his analysis cannot be taken as a complete design method even for the typical interior panel considered.

However, since for the special case considered by Nichols, the shear along the panel boundaries is in fact zero, the moments acting along these boundaries will be maximums. The total moment in either direction can be assigned arbitrarily between the parallel boundaries and the resulting moment pattern will always satisfy the lower bound requirement that no moments inside the panel will exceed the boundary moments. This allows for the selection of reinforcing based on boundary moments alone and, in addition, the termination of reinforcement inside the panel may be found from equilibrium.

Based on this principle an equilibrium method, known as the Direct Design Method, was introduced in North America with the adoption of the 1971 ACI code(3). This method is intended for use in the design of slab systems, with or without beams, having fairly regular layouts and acted on only by uniformly distributed, vertical loads. These limitations were considered necessary so that the specified critical section would closely resemble a segment bounded by zero shear lines and that the rules given for assigning

moments would not result in problems with serviceability.

The Direct Design Method involves three fundamental steps.

i) determination of the total factored static moment for the panel

ii) splitting of this total factored static moment to negative and positive design moments located at the edges of the critical section, ie. face of the columns and midspan respectively.

iii) distribution of the negative and positive design moments along the critical section to the column and middle strips and to the beams, if any.

The recommendations used to split the static moment into negative and positive components and then distribute these into column and middle strip moments are based on extensive test results and are intended to approximate an elastic distribution. This helps minimize any problems due to excessive cracking or deflection.

Since the Direct Design Method is based on the satisfaction of equilibrium and the use of critical sections approximating lines of zero shear, it can be thought of as approaching a lower bound solution.

The first practical lower bound design method was developed in Sweden by Arnie Hillerborg(1) and is known as the strip method of design. In this method the slab is reduced to one-way strips spanning in each direction. Each strip is considered to act independently much as a

continuous beam, carrying an appropriate portion of the total applied load. The basis for this approach can be seen by neglecting the twisting moment term in the plate equilibrium equation below(4). The resulting expression can be uncoupled into two beam equations.

$$\frac{\partial^2 m_x}{\partial x^2} + 2\cancel{\frac{\partial^2 m_{xy}}{\partial x \partial y}} + \frac{\partial^2 m_y}{\partial y^2} = q_x + q_y \qquad (1.2)$$

The load is carried by these strips in either of the principal directions and the resulting, resisting moments calculated as for beams. This procedure is valid only for those support conditions for which the effects of twisting moments can be neglected or accounted for independently. This occurs when each design strip has sufficient supports to be considered as a stable beam. For slabs supported on columns, for example, this is not the case and modifications are required.

To account for concentrated loads and for point supports in a slab which could otherwise be solved using the simple strip method, Hillerborg introduced the concept of a load dispersion element which converted the point load to a patch load, distributed uniformly over an assigned area. This distributed load was then combined with any other distributed loads and moments computed using the simple strip method. To satisfy lower bound theorem requirements he developed an approximate moment field for the dispersion element, loaded with the patch load acting in the opposite direction to the point load or support. This moment field

was superimposed onto those moments obtained from the strip method to give a complete lower bound solution.

The concept of using a load dispersion element works well for panels supporting one or two point loads of known magnitude. For panels with point supports the method becomes unworkable in practice in that a small variation in the assumed magnitude of the support reaction can lead to large variations in resulting moment fields. A trial and error procedure to obtain reasonable moment fields is quite tedious.

To overcome this situation Hillerborg(1) introduced the concept of elements bounded by lines of zero shear. This procedure was named the *Advanced Strip Method* by Crawford(5).

Hillerborg introduced three types of elements with the presentation of his advanced strip method.

a. Type I: rectangular segment supported along one edge; transfers load in one direction only

b. Type II: triangular segment supported along two edges, transferring load in one direction

c. Type III: load dispersion or corner supported element transferring load in two perpendicular directions.

A rectangular, simply supported slab with a center column is shown in Fig. 1.2 to illustrate the division into elements.

The basis behind the advanced strip method was to satisfy equilibrium in each of the elements and then

ensuring compatability of moments across common element boundaries, piece the segments together to give a complete moment field for the slab. Hillerborg also presented some practical guidelines to be used in distributing these moments which would permit simple steel layouts and also ensure adequate serviceability.

The major drawback to both the simple strip and advanced strip methods is that they are unsuitable for flat plate construction, even with regular layouts. Recognizing this fact Hillerborg published a book in 1974 in Swedish, (translated to English in 1975)(6), in which he presented his strip methods in detail but also introduced a rationalized design procedure for regular column supported flat slabs. The method divides the slab into a number of essentially Type III segments. Standardized, preset moment fields are applied to the individual elements to obtain a complete moment field solution for the slab. These element moment fields are designed so that:

- the individual element is in equilibrium
- equilibrium is maintained between adjacent elements
- a uniform positive moment can be superimposed on the element
- the edge moments are not exceeded anywhere within the field, ensuring a lower bound solution.

Hillerborg's rationalized design procedure was not applied to flat plates with irregular column positions and no recommendations were given for treating these types of

problems. In essence this rationalized element procedure is quite similar to the DDM, differing only in certain details of distributing moments. For the regular slabs for which it is applicable there is little advantage in using this approach over the Direct Design Method.

Although not much used outside Sweden, there is a considerable amount of discussion and application of Hillerborg's work in the literature(5)(7)(8).

One other plastic design approach which should be mentioned here is Wiesinger's(9) segment equilibrium method, developed for flat plates with very irregular column layouts. It is based on assuming as lines of zero shear the lines joining the column centers, the lines forming the right bisectors of these lines and the lines joining the columns to the intersection of the bisectors. The result is a series of right angled triangles supported at one of the acute angles by a column. Segment moments are obtained by summing moments about orthogonal axis passing through the supported corner and parallel to the perpendicular edges of the triangle.

The total segment moments are proportioned between negative and positive moments. A uniform bottom mat is placed over the entire slab to resist the positive moments and the negative moments, from all segments meeting at a support, are resolved into two convenient orthogonal directions and an isolated top mat provided to resist them.

Unlike Hillerborg, Wiesinger did not attempt to furnish a complete moment field so his solution cannot be considered as lower bound.

The Segment Design Method presented in this paper is not a new concept but a continuation of the work reviewed in this section. The purpose is to develop a practical procedure for the design of flat slabs on both regular and irregular column layouts. Hillerborg's theory behind the corner and edge supported elements is borrowed from heavily and the idea of a logical progression of steps leading to a distribution of moments over the slab is in fact an adoption of the philosophy used in the Direct Design Method.

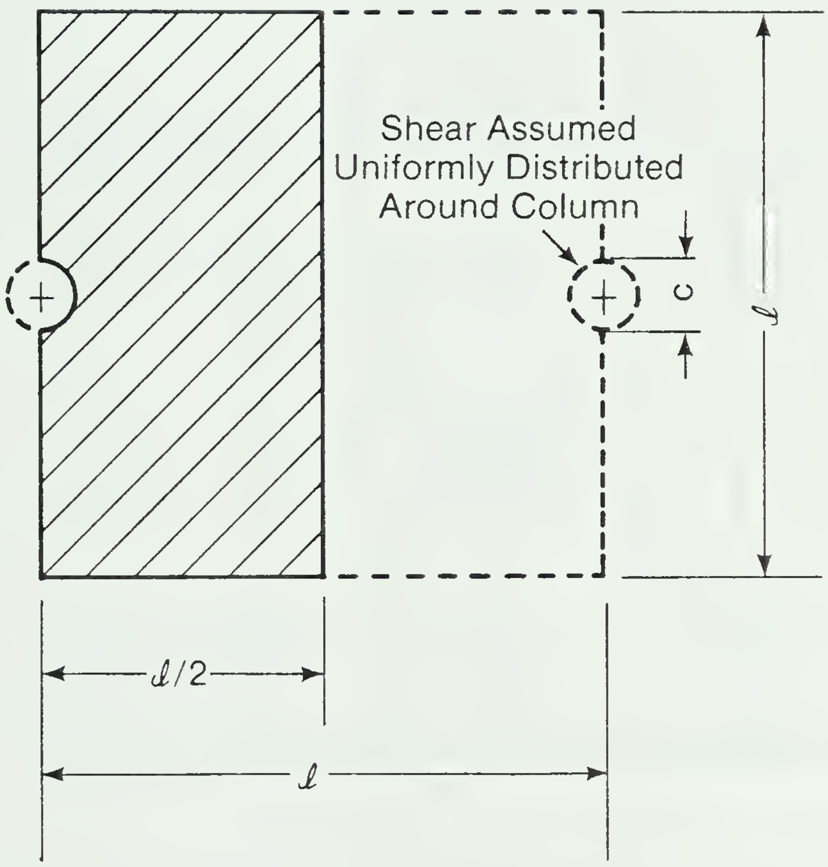


FIGURE 1.1 Slab Section Used by Nichols

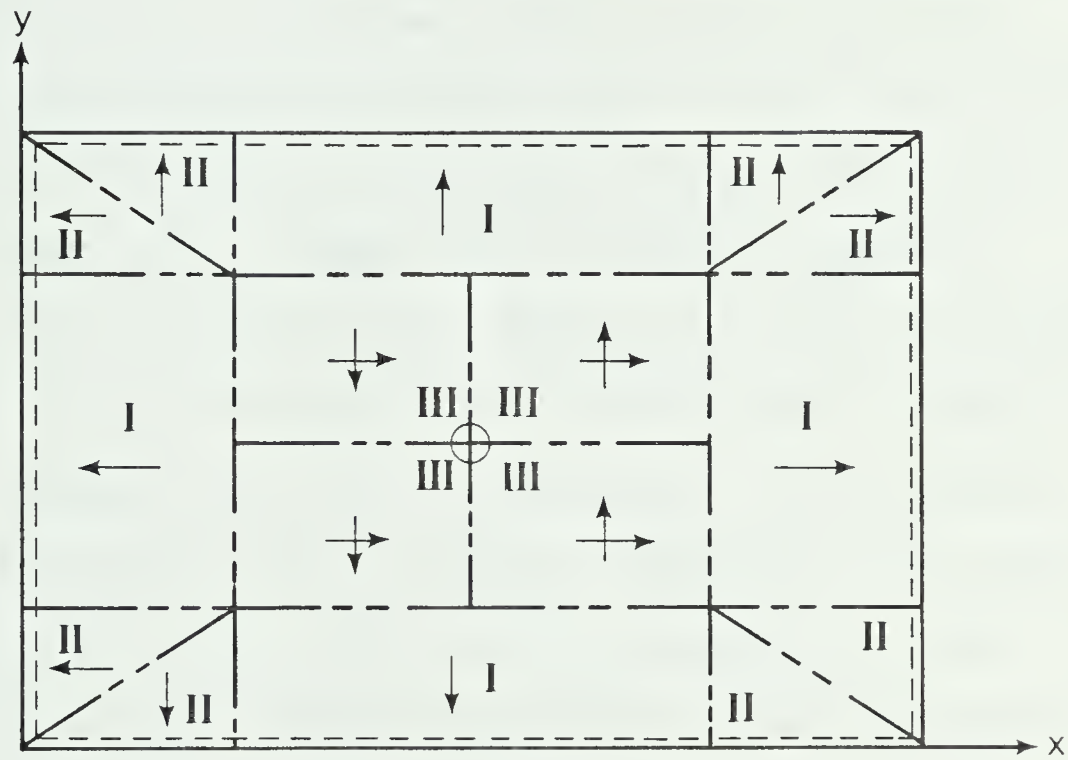


FIGURE 1.2 Division of Rectangular Slab into Elements

2. The Segment Design Method for Reinforced Concrete Slabs

2.1 Introduction to the Segment Design Method (SDM)

The Segment Design Method (or SDM) was developed to provide the design engineer a means of producing safe and economical slab systems under practical design office conditions. It is being offerred as a possible alternative to the present code procedures, being simpler than the Equivalent Frame Method and more versatile than the Direct Design Method. The SDM is a procedure whereby the entire slab is divided into a number of rectangular segments bounded by lines of zero shear. Moments are calculated along the boundaries so that equilibrium is obtained for each individual segment and there is compatability of moments across the boundary lines of adjacent segments. The total slab design arises from the combining together of the predetermined moment fields from the individual segments to give a total moment pattern for the slab system.

The basic assumption for producing serviceable lower bound slab designs is that if a simplified moment field can be produced for each segment which is lower bound and also has a distribution similar to an elastic moment field, for the same segment, then not only will equilibrium be satisfied but serviceability should be ensured as well.

Economy of design is achieved by using very simple reinforcement layouts. The steel used to resist positive

moments is selected to provide a uniform bottom mat throughout the slab which has a lower limit controlled by the requirements of shrinkage and temperature reinforcement. To resist the negative moments top mats are placed over the columns and are isolated to these areas. This type of distribution has been recommended by Wiesinger(9) and Hillerborg(1) and has proven effective in tests by Cardinas and Kaar(10).

There are three types of segments associated with the Segment Design Method which are similar to, but used quite differently from, the three elements used by Hillerborg in the advanced strip method. These are given below with their appropriate forerunner from the advanced strip method given in brackets.

- Corner Supported Segment (Type III element)
- Edge Supported Segment (Type I element)
- Adjacent Edge Supported Segment (2 Type II elements).

The first and most commonly used of the segments is the corner supported segment shown in Fig. 2.1. The segment is bounded by zero shear lines with a uniform positive moment along the edges opposite the support. The area of negative moment is controlled by the values of α and ϵ.

Figs. 2.2a and 2.2b are the edge supported and adjacent edge supported segments, respectively. The notation used for these is similar to that for the corner supported segment and needs no further explanation.

Section 2.2 lays out the methodology of the Segment Design Method in step form so that there is a concise direction to the design. The remainder of the chapter deals with the specifics in the development and reasoning behind each of the individual steps.

2.2 Methodology for Segment Design Procedure

2.2.1 The Design Method

The steps in the design process are as follows:

i) select a trial thickness for the slab

ii) choose trial positions of segment boundary lines or lines of zero shear, (thus defining necessary segments)

iii) calculate the total static segment moments in each segment

iv) split the total segment moment into components of negative and positive moments, using the requirements of minimum positive steel and the recommended α values as guides

v) distribute the edge moments in accordance with the provisions outlined in section 2.2.4; modify moments by repeating steps iv) and v) if necessary

vi) evaluate the trial slab thickness on the basis of the shear-moment interaction at the supports

vii) select the reinforcing steel for the top mats and

detail the layout and cutoff points of the bars.

Only steps ii) to v) are unique to the SDM as the other steps are common to all slab design procedures. As a result only these four steps will be discussed in detail in the thesis.

2.2.2 Positioning of Segment Boundary Lines

In order to have maximum moment at the edges of the design segment, the segment boundaries must be lines of zero shear. It is assumed that these shear lines are straight lines, forming rectangular segments and that the segments cover the entire slab.

To aid the designer in positioning these lines certain basic rules are presented which will generally result in a good first approximation. For irregular slabs, final positioning of the segment boundary lines may require a trial and error procedure in order to obtain a satisfactorary moment field.

First, boundary lines should be selected to coincide with any axis of symmetry since these, by definition, are lines of zero shear. From this it follows that for regular, continuous panels, with equal spans in both directions, the boundary or zero shear lines will occur at the panel midspan and column centerlines. Even when the panels are not equal in span, this positioning of the boundary lines is recommended, since it can be shown that a lower bound solution may be obtained for any reasonable assumed position

of the zero shear lines.

If the columns are idealized as point supports the segment boundaries pass through the column. If the columns have significant dimensions it is known that the total panel moments will be reduced somewhat due to the reduced effective span. The Direct Design Method accounts for this by taking the critical section for negative moment at the face of the equivalent rectangular support. It is suggested that the same procedure be used here, that the segment support edge in each direction be taken as passing through the face of the equivalent rectangular support. This results in slightly different segments in each direction as shown in Figs. 2.3a and 2.3b, but in each case the total load between supports in a given direction is accounted for.

For panels that are not continuous in a given direction, ie. exterior panels, the position of the zero shear line in the positive moment region will not necessarily be at midspan. An initial approximation to the position of this line may be obtained by assuming the slab to parallel the behaviour of a continuous beam. For slabs with very flexible columns and no beams the line of zero shear would approach 3/8 of the panel length from the edge. This value will increase to 1/2 the length for a very stiff system of exterior edge support.

To evaluate the above approximation a series of slab configurations were analyzed using the finite element program *Hybslab*(11) with varying ratios of exterior to

interior column stiffnesses. These analyses provided elastic moment fields for the slab from which the points of maximum positive moment could be found. A plan view of the slab geometry used in the analyses is shown in Fig. 2.4.

To determine the effect of the ratio of exterior column to interior column stiffnesses, k, on the position of the zero shear line in an exterior panel, analyses were carried out on slabs with k values varying from 0.1 to 1.25. Figs. 2.5 and 2.6 are contour plots of the x-moments from the analyses for k equal to 0.5 and 1.0 respectively. As shown the position of the zero shear line moves very little over a relatively wide range of k, being at 0.42 of the span length for k equal to 0.5 and at 0.45 of the length for k equal to 1.0. Other values of k were found to give basically the same results. Based on these results it is recommended that the fraction of the panel given to the exterior segments be taken within the range of 0.42 to 0.45 for most cases. Also indicated by Figs. 2.5 and 2.6 is the validity of assuming the zero shear line as a straight line.

For slabs with walls where edge supported or adjacent edge supported segments are required the following positions of zero shear lines are recommended. For an exterior edge supported segment the line of zero shear will fall within the range of 0.40 to 0.50 of the panel length. In general a value of 0.50 can be assumed with reasonable accuracy for fully supported edges.

For elements used as adjacent edge supported segments the lines of zero shear are dependent on the segments adjacent to that element (see Fig. 3.15), especially in the case of irregular slabs. No general recommedations can be made and frequently a trial and error procedure is required. When selecting the position of zero shear the designer should remember that positive moments across adjacent segment boundaries must be continuous.

It is suggested that once the positions of the boundary lines are selected they be shown on a scaled plan drawing of the slab. This can also be used for the calculations, and simplifies considerably the evaluation of the design.

2.2.3 Calculation of Segment Moments

For each corner supported segment the total static moment, M^o is determined by summing moments about an axis through the face of the support in each of the x and y-directions. The calculation of these moments, in the x and y-directions respectively, are independent of each other therefore it is possible to use one set of segments for moments in one direction and another set of segments for moments in the opposite direction. The argument here is based on the fact that there is no one unique solution. If a set of segments is chosen there exists a lower bound solution for moments, in say the x-direction, for those particular segments. A different set of segments in the same slab also provides a lower bound solution for the moment

fields in that direction. A similar reasoning applies to moments in the y-direction hence one may select moments in one direction from the first set of segments and the moments in the opposite direction from the second set of segments without violating slab equilibrium and still have a lower bound solution for the entire slab. This will be illustrated with the calculations in Example II.

For the corner supported segment shown in Fig. 2.1, the total segment moments in the x and y directions respectively are;

$$M^o_x = 1/2 q s_x^2 s_y \tag{2.1}$$

$$M^o_y = 1/2 q s_y^2 s_x \tag{2.2}$$

The edge supported segment in Fig. 2.2a, having a reaction only along the y-axis has the load carried by moments in the x-direction only. Summing moments about the support results in a total segment moment of:

$$M^o_x = 1/2 q s_x^2 s_y \tag{2.3}$$

A similar equation could be written with the subscripts reversed if the support was along the x-axis.

Note that although no stated segment moment is computed in the direction parallel to the support, a linear distribution of moments in this direction may exist due to

moments applied along the edges perpendicular to the support from adjacent segments.

Due to the static indeterminancy of the adjacent edge supported segment it is not possible to write moment equilibrium equations about the supported edges without introducing certain assumptions. Originally it was decided to assume a shear distribution along the adjacent supported edges. In practice this proved to be quite useless. It was found, similarly to Hillerborg's load dispersion element, that a very small change in the shear distribution along an edge may cause a very drastic change in moments. No recommendations for design could be given using this approach.

A second approach to the problem, and the one finally settled on, was to assume a line of zero shear extending from the supported corner across on the diagonal to the opposite corner as shown in Fig. 2.2. This enabled the segment to be broken into triangular regions, not unlike the Type II elements used in the advanced strip method, each carrying the load to their respective edge support. The resulting expressions for the static moment on the triangular regions are given by equations 2.4 and 2.5.

$$M^o_x = 1/6 q s_x^2 s_y \qquad (2.4)$$

$$M^o_y = 1/6 q s_y^2 s \qquad (2.5)$$

While the above equations are similar to those written for Hillerborg's Type II elements the use of the moments in the SDM is quite different in that they apply to the entire rectangular segment and a complete moment field, including possible twisting moments is evaluated.

2.2.4 Selection and Distribution of Positive and Negative Moments

The total static segment moments must be allocated to the parallel boundaries as positive and negative moments. As will be shown in chapter 3 any practical distribution between positive and negative moments, for any segment, will result in a lower bound moment field, (ie. maximum moments occur at the edges).

As a starting point an initial uniform positive moment field is selected for the entire slab, which would correspond to the moment capacity of the shrinkage and temperature reinforcement. In the case of heavy loading this can be increased. The negative moments are calculated for each segment from equation 2.6

$$M^{-}=M^{o}-M^{-} \tag{2.6}$$

The parameter ψ, defined as the ratio of negative, M^{-} to positive edge moments, M^{+}, in a given direction may be used as a basis for determining the suitability of the

positive and negative components calculated.

The recommended values of ψ are based on meeting serviceability requirements in that they are selected so as to approximate the results from an elastic moment field, which will generally provide adequate serviceability. The results from the analytical analyses done on the slab in Fig. 2.4 were used to calculate ψ values for two segments in an exterior panel for different relative stiffnesses between exterior and interior columns. The results are plotted graphically in Fig. 2.7. As expected, the ratio ψ decreases with a decreasing ratio of exterior to interior column stiffness for both the exterior and first interior segments.

For the usual case, when the k ratio is between 0.75 and 1.0, the exterior ψ ratio is within the range of 0.40 and 0.45. For the same range of k the results show ψ for an interior segment to be approximately equal to 1.5. Allowing for some amount of moment redistribution it is recommended that ψ be within the range of 1.35 and 2.0 for interior segments. These values are recommended for serviceability criteria only. It should be noted that any deviation from the guidelines will not in any way affect the validity of the application of the lower bound theorem to the segments.

With the positive and negative moments known and indicated on a plan view of the slab it is now up to the designer to decide if there is an adequate set of moments to proceed. Of the things that can be checked at this point the designer should ensure that there is continuity of moments

moments between adjacent segments, that there is not too large an unbalance at any of the support edges, and that the ψ values fall within the recommended ranges given above.

If the ψ values calculated fall outside the suggested guidelines the designer has two options open to him. First if ψ is too small he can increase the negative segment moments to obtain a desired ψ, in which case the positive moment steel is not considered fully effective on the basis of strength, or if ψ is too large the designer can either reduce the negative support moments or increase the amount of bottom steel to 'bump up' the positive moment. In no case is the bottom reinforcement specified to be less than the requirements for shrinkage and temperature steel.

A second option, if it is felt that adjusting the ψ values will not give a satisfactorary moment field, is to move the segment boundary lines thus changing the total segment moments.

The distribution of negative moments is dependent on the type of segment being considered. The support moments for the corner supported segment are distributed uniformly over αs_x and ϵs_y. The values of α and ϵ should be such so as to prevent a flexural punching failure from forming at the columns. Recommended values to prevent this type of failure are given in chapter 3.

Negative moments on an edge supported segment are to be uniformly distributed across the support span resulting in a uniform bottom mat of steel. For the case where the support

edge is simply supported no top mat is necessary.

The adjacent edge supported segment is a special case where top steel is required whether or not the support conditions are fixed or simply supported in that with the simply supported case there are significant twisting moments in the support region which must be accounted for. This will be explained further in chapter 3.

To assist in the selection and spacing of reinforcing, to resist the design moments, Tables A.1 and A.2 in Appendix A, give moment capacities for various bar spacings and slab thicknesses for numbers 10 M and 15 M bars respectively. When calculating the cutoff points for the top mats simplified moment fields such as shown in Fig. 3.8 may be used as a guide to determine where the steel is no longer required to resist the negative moments.

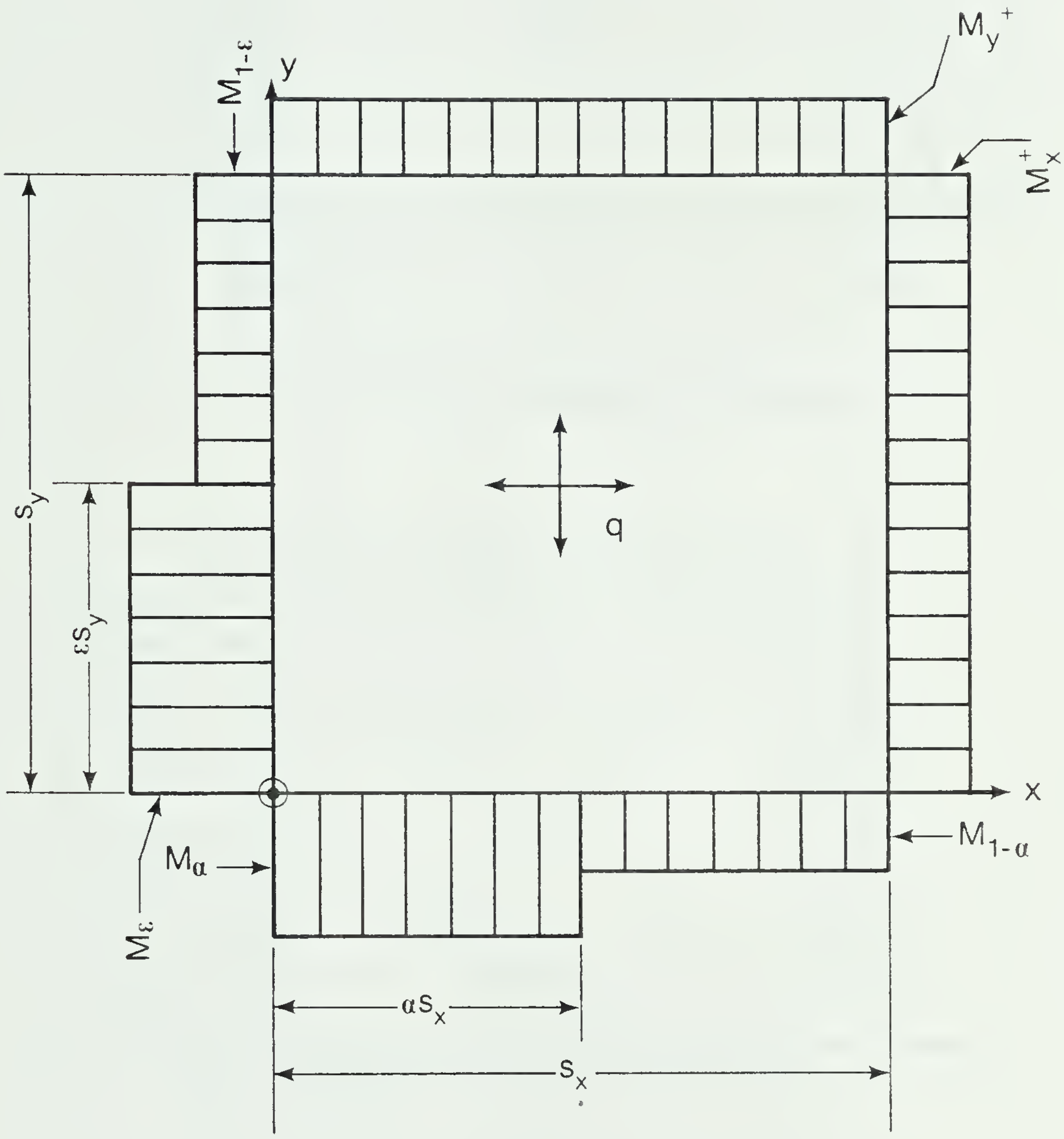
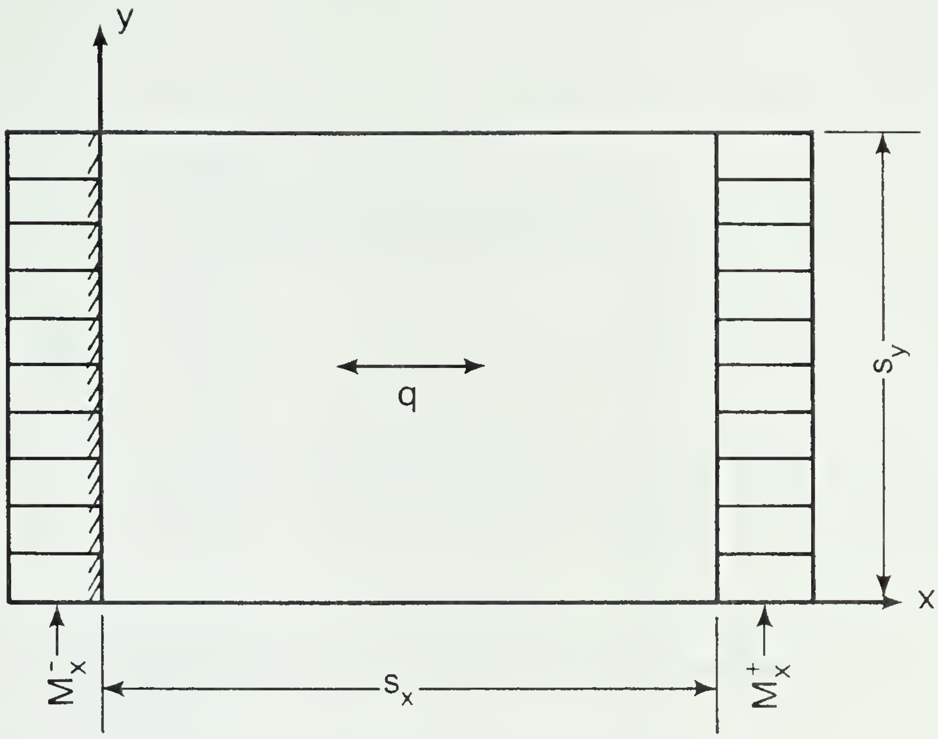
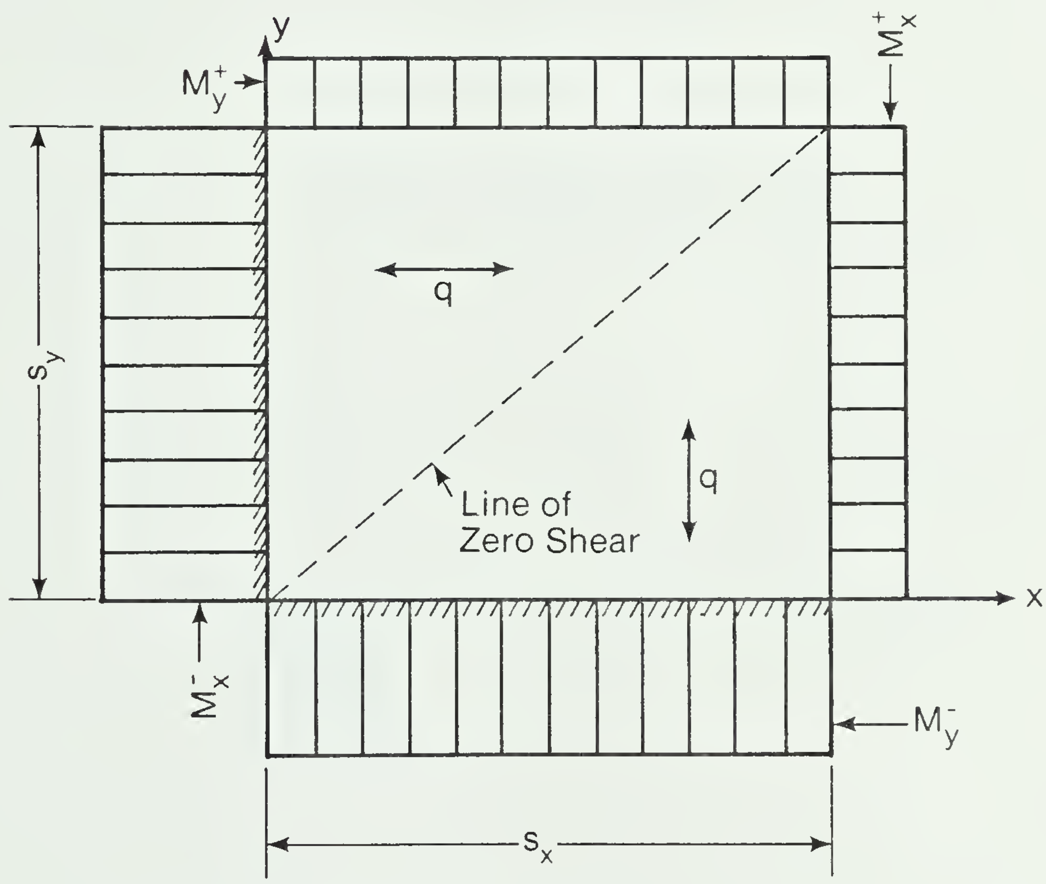


FIGURE 2.1 The Segment Method's Corner Supported Segment



a. The Edge Supported Segment



b. The Adjacent Edge Supported Segment

FIGURE 2.2 The Edge Supported and Adjacent Edge Supported Segments

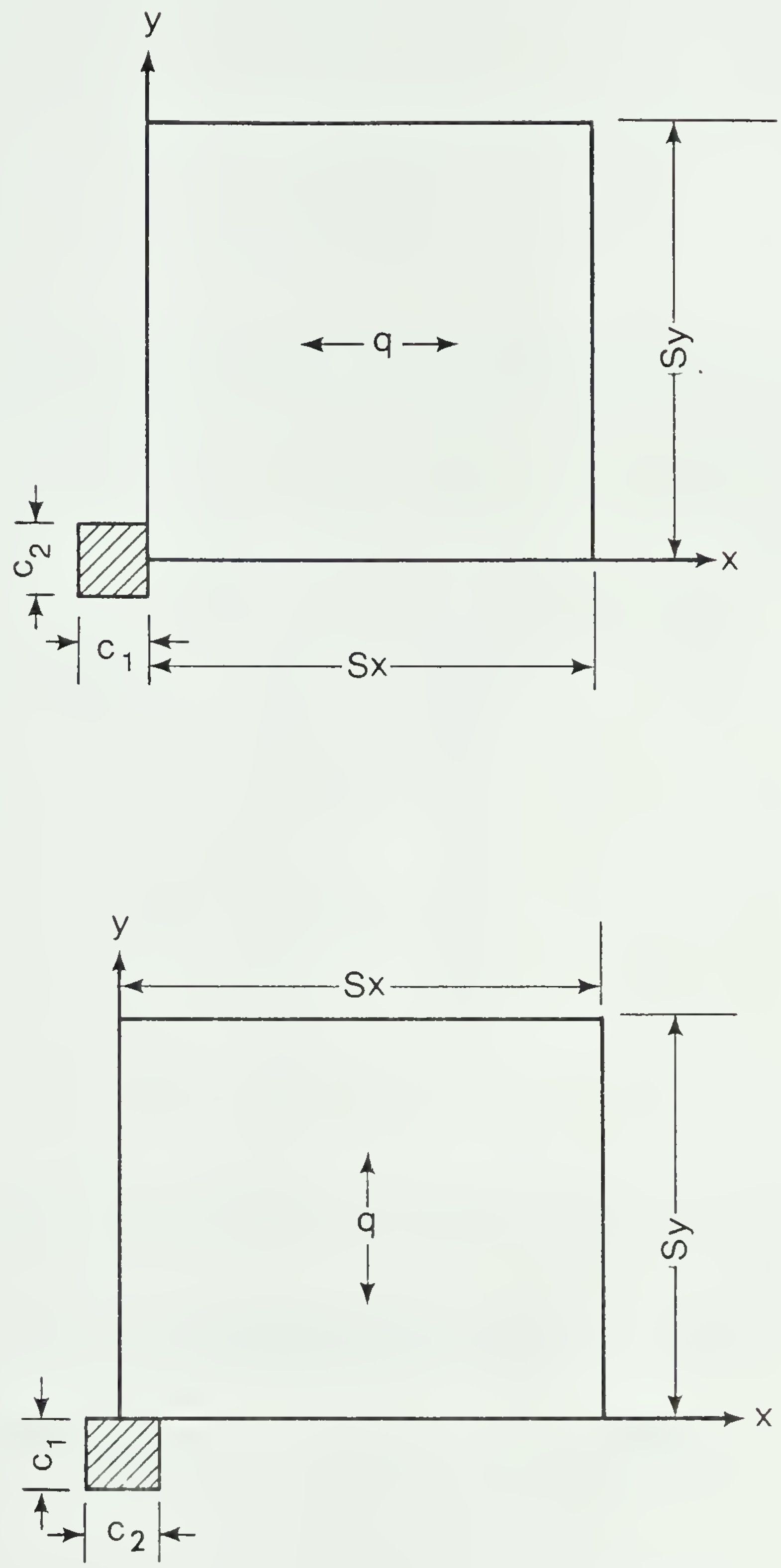


FIGURE 2.3 Effect of Column Size on Boundary Dimensions for a Corner Supported Segment

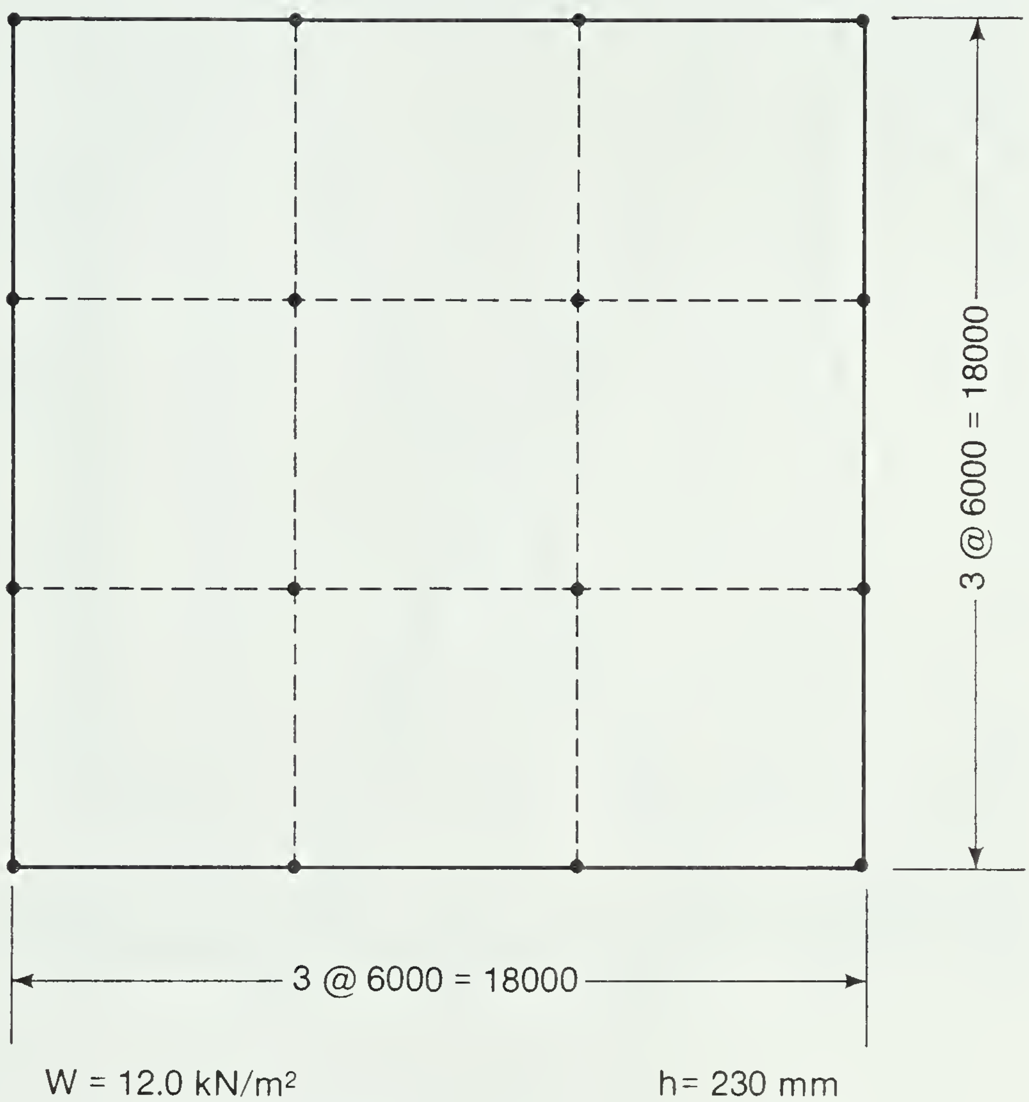


FIGURE 2.4 Slab Configuration Used for Analytical Tests

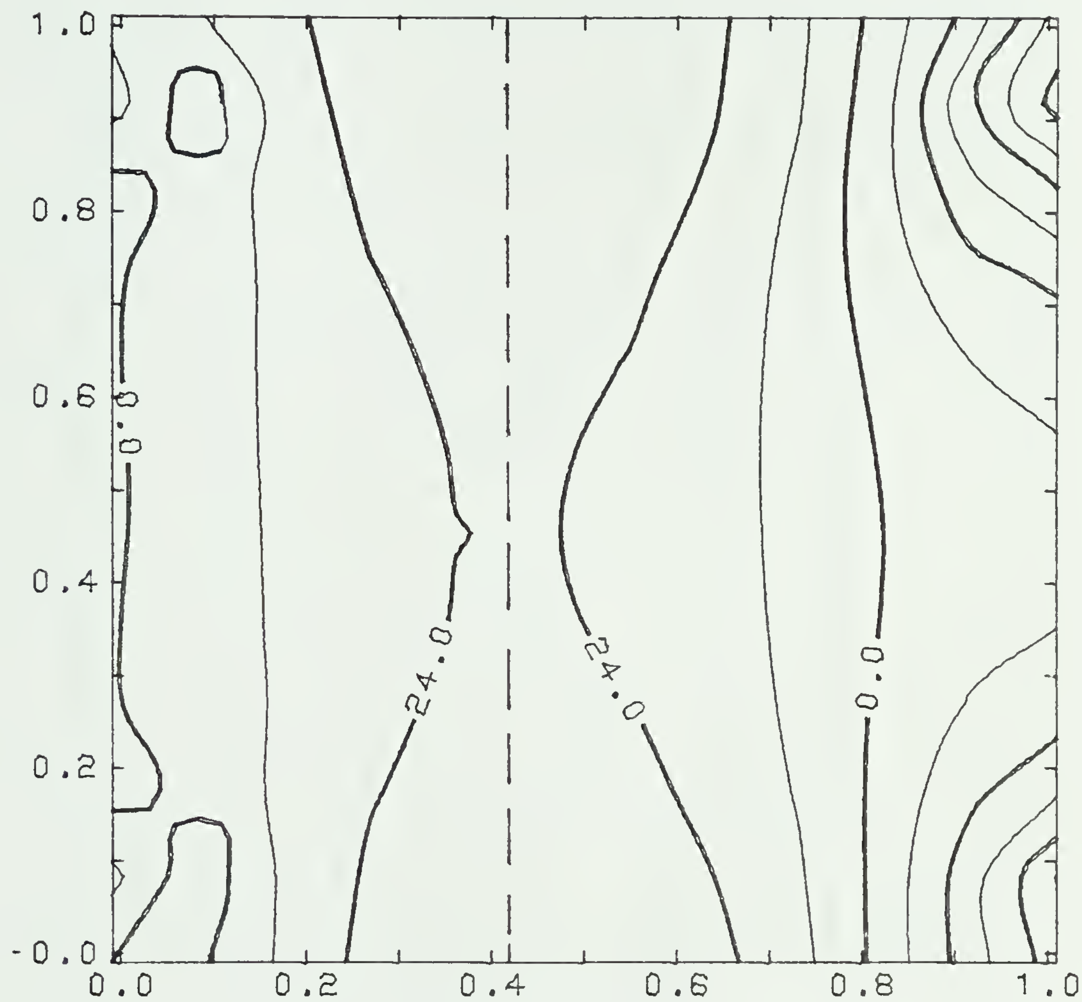


FIGURE 2.5 Position of Assumed Zero Shear Line for k=0.5

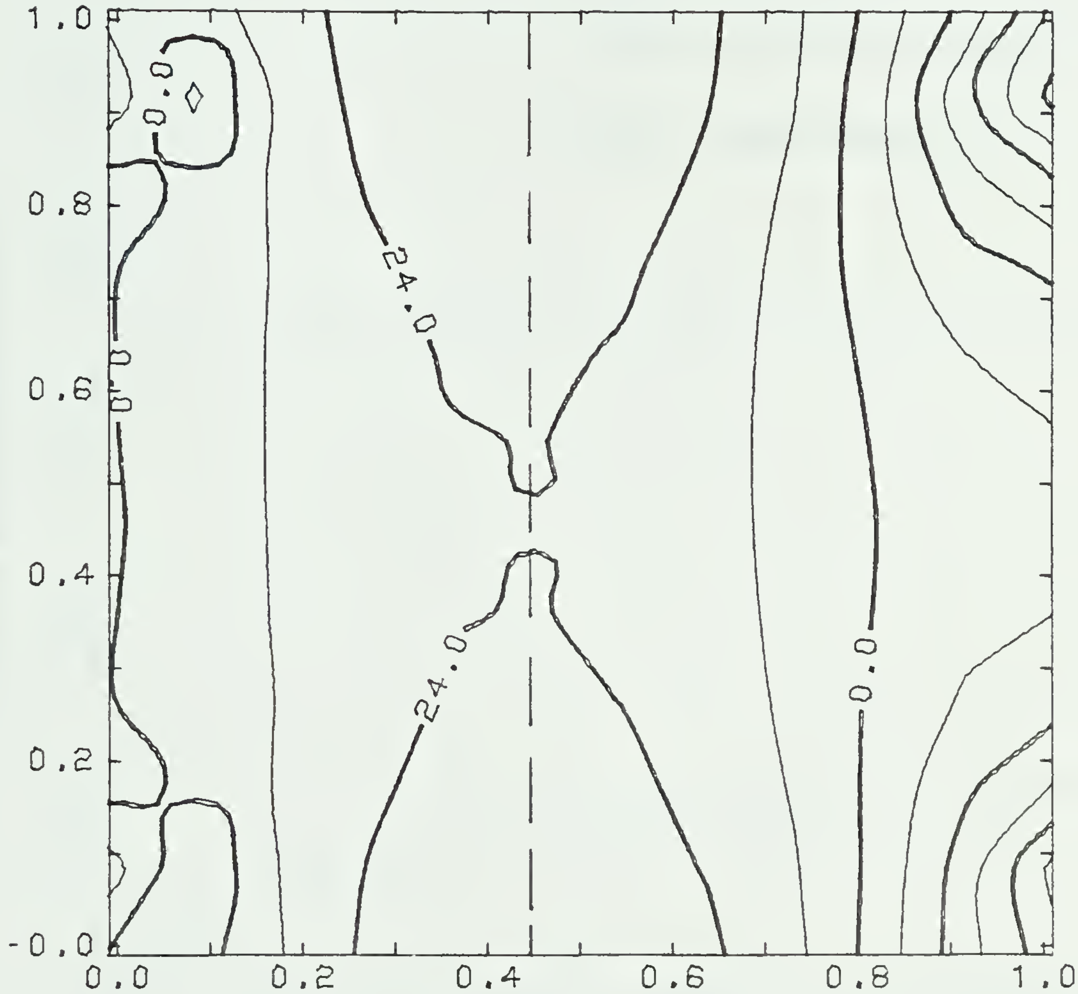


FIGURE 2.6 Position of Assumed Zero Shear Line for k=1.0

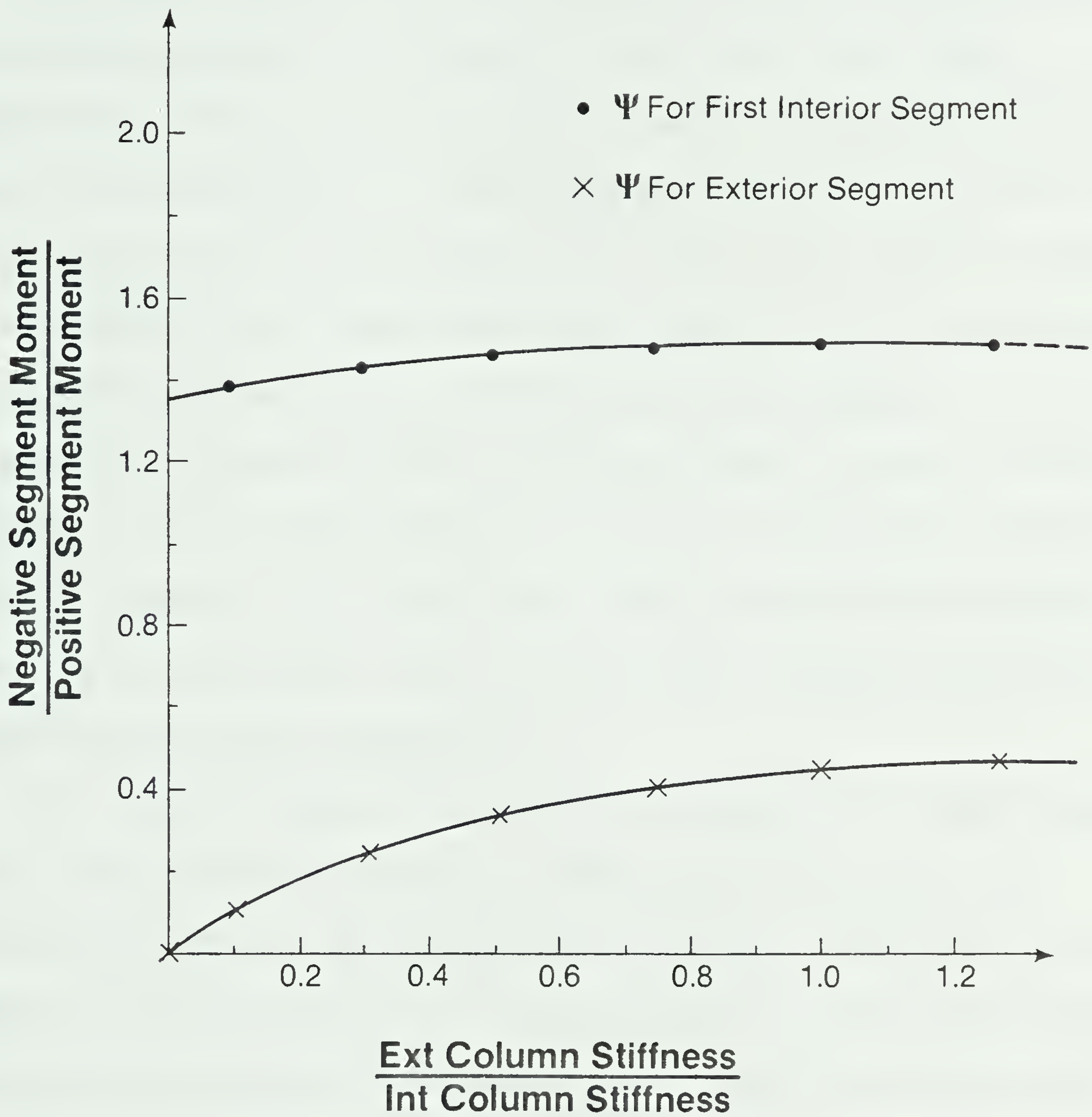


FIGURE 2.7 Plot of Relative Column Stiffnesses to Ratio Ψ

3. The Properties of Segments

3.1 Segment Development

The main elements in the Segment Design Method(SDM) are the segments used to divide the slab into individual components. The procedure used to obtain continuous moments along the boundaries of these segments and still satisfy the total equilibrium requirements has been described in chapter 2. To ensure that lower bound requirements are satisfied and to provide guidance for curtailing reinforcement, it is necessary to know what the internal moment fields for each segment will be. Knowing the nature of this distribution of internal moments will also facilitate reasonable boundary moment distributions, particularly for irregular spacing of supporting columns and walls.

For these reasons the internal moments for each segment type, for different boundary moment distributions are examined in detail by obtaining solutions using the finite element program *Hybslab*(11). By comparing the elastic moment fields for different boundary moment distributions within a segment some insight can be obtained into the proper selection of these boundary moments so that a safe and serviceable design can be achieved.

The chapter is divided into three sections, each devoted to a particular segment. Beginning with the corner supported segment each is presented individually with a

detailed explanation of its particular properties and characteristics. The edge supported and adjacent edge supported segments follow respectively. The results from the analysis of each segment, for particular boundary moment distributions applied to it, can be used to derive simplified moment fields which can be used in design (see Fig. 3.8).

3.2 The Corner Supported Segment

The corner supported segment used in the Segment Design Method is an adaptation of Hillerborg's(6)(5) Type III element. The segment is rectangular in shape and supported essentially at one corner only. Except along the column faces (see Fig. 2.3), the four edges of the segment are bounded by assumed lines of zero shear. The distribution of boundary moments chosen for the segment correspond to those shown in Fig 2.1.

Since for a corner supported segment the negative moment is concentrated near the support it would seem logical then to have M^-_x act only over a length of ϵs_y and M^-_y act over αs_x. The segment and resulting distribution of moments is as shown in Fig. 3.1. This is consistent with the decision to provide isolated top mats providing negative moment resistance.

To provide some guidance in selecting α and ϵ values a series of elastic analyses were carried out on a corner

supported segment with an aspect ratio equal to one, ψ equal to 1.5 and varying α and ϵ. The resulting moment fields were examined and comparisons made of the positions of the zero moment lines. It has been found that although some leeway is possible, the best results in terms of approximating an elastic solution for a complete slab is that the negative moments be distributed with α and ϵ within the range of 0.45 to 0.6.

Although theoretically these values may vary from 0 to 1.0 it is recognized that failure at the column may occur from the development of a yield fan extending out laterally from the support, hence, it is further recommended that α and ϵ not be taken less than 0.45 of the representative segment length in interior segments and not less than 0.5 for the exterior segments. Higher values of α and ϵ would not be overly conservative. In cases where two corner supported segments having highly different areas share a common edge boundary it may be necessary to distribute the negative moments in the smaller segment based on a large α or ϵ to maintain continuity across the support edge. In this case α and ϵ take on values based on the larger segment length.

Elastic moment fields were also found for corner supported segments with varying aspect ratios. In all cases α, ϵ and ψ were held constant. The segments tested had a long span of 3.0 m and were acted upon by a uniform design load of 12.0 kN/m^2. A comparison was made of the location of

zero moment for segments with aspect ratios varying from 0.5 to 2.0. The results of this test indicate that the basic distribution of moments is independent of the aspect ratio. This is seen for aspect ratios of 0.5 and 2.0 for the corner supported segment shown in Fig. 3.2, where the contour lines represent the x-direction moments in kN-m/m.

The position of the point of inflection is affected by the ψ value however. Plots of the zero moment line for the normal range extremes of ψ between 0.5 and 2.0 are shown in Fig. 3.3. The same segments analyzed under varying ψ values with all other variables constant indicate that for ψ equal to 0.5 the point of inflection is at approximately half the distance as it is for ψ equal to 2.0. Again the contours represent moments in kN-m/m and the segment span length is 3.0 m. It is suggested since the majority of corner supported segments are interior segments with high ψ values that the position of zero moment be taken between 0.4 and 0.45 of the segment length in all cases.

To determine the complete distribution of moments within a segment an elastic analysis was carried out on an example segment subjected to a uniform load of 12.0 kN/m^2 with α and ϵ equal to 0.5, the aspect ratio equal to 1.0 and ψ equal to 1.5. The segment length was 3.0 m. The x-direction moments resulting from this analysis are indicated by a typical, three dimensional profile of the elastic moment field, for a corner supported segment shown in Fig. 3.4. Fig. 3.5 is a contour map of the x-moments on

the segment where the contours represent moments in kN-m/m. A similar distribution holds for the y-direction moments.

From the profile map it can be seen that nowhere inside the segment are the edge moments exceeded. This supports the lower bound theory and allows the designer to base his design on the edge moments alone, knowing that strength-wise the design is on the safe side. The contour plot further substantiates the results of Figs. 3.2 and 3.3, that the zero moment line is between 0.4 and 0.45 of the span in the direction that moments are being considered.

Hillerborg(6), in developing his rationalized load dispersion element, made it clear that the moments along an edge should be of the same sign. However, in using the Segment Design Method for the design of slabs with irregular column layouts, it has been found that in certain cases continuity requires moments of opposite sign on an edge. This condition has been analytically tested using the elastic model and the results show that the lower boundness of the segment is not affected so long as a suitable distance is given for the transition from negative to positive moment along the support length. It is suggested that a minimum transition of 0.20 of the edge length be provided for this purpose. Fig. 3.6 shows the distribution of edge moments for a 3.0×3.0 m square segment tested with α and ϵ equal to 0.45 under a uniform design load of 12.0 kN/m^2. Fig. 3.7 shows the profile of moments for such an edge distribution indicating that the edge moments are still

lower bound.

In developing a simplified version of the moment field for the corner supported segment the elastic moment fields were used as a means of determining the lines of inflection, and by approximating the elastic moments the segment will generally satisfy all serviceability requirements. Based on Figs. 3.4 and 3.5 a more simplified moment field, for a general segment with l/l in the order of 1.5 and 2.0, is presented in Fig. 3.8. This simplified moment field can be used to calculate the theoretical cutoff points for the reinforcing. The steel embedment lengths as specified in clause 10 of CSA-A23.3(12) must be added to these lengths to obtain the actual cutoff points. The advantage to using the simplified moment fields, especially with irregular support layouts, is that if one had a family of such plots for different l/l values they could be overlain on a plan of the slab resulting in a complete design moment field for the slab.

With the background information gained from the test analyses the corner supported segment, when used with the recommended distribution of edge moments is safe with regard to strength requirements and will provide adequate serviceability.

3.3 The Edge Supported Segment

The edge supported segment is supported along one edge with the other three sides bounded by assumed lines of zero shear. The load is carried to the supported edge in the perpendicular direction with no load assumed transferred parallel to the support. Moments are summed about the supported edge to obtain the segment static moment. To maintain continuity between segments a uniform positive moment is superposed on the edge opposite the support. The negative moment is uniformly distributed along the support edge and the sum of the total negative and positive moments make up the static moment. For a simply supported edge segment the total moment is carried by the positive moment at the span edge only. The distribution of moments for an edge supported segment with a moment resisting support is shown in Fig. 2.2.

A linear distribution of moments is assumed between the edges adjacent to the support to satisfy continuity of any moments from adjacent segments.

A series of elastic analyses done on a typical segment under uniform load indicate little or no effect in the distribution of moments under varying degrees of rectangularity. Figs. 3.9a and 3.9b indicate the position of the line of zero moment for aspect ratios of 0.5 and 2.0 respectively. It has been found that, in either the long or short direction, the pattern of moments resembles that of Fig. 3.10 where the point of inflection varies between 0.375

and 0.50 of the span length. The elastic moment distribution for a typical edge supported segment of 3.0 m span, under a uniform load of 12.0 kN/m^2 and with ϕ equal to 1.5, is illustrated by a contour map in Fig. 3.11. From Fig. 3.10 it is evident that the moments anywhere within the segment are easily calculated.

3.4 The Adjacent Edge Supported Segment

The adjacent edge supported segment was developed to enable the designer to be able to calculate the moments in an area bordered by two walls perpendicular to each other. For slabs continuously supported along the edges with walls, for example water reservoirs, it is a convenient way to handle the corner areas. As will be demonstrated it can also be used in combination with the corner and/or edge supported segments to find moments either around interior partitions or along slab edges with discontinuous supports.

As with the other segments the total moment is split into positive and negative components. The positive moments are distributed uniformly along the edges opposite the support edges (ie. on the half of the segment not accounted for in the equilibrium equations). This is illustrated in Fig. 3.12 for a typical segment along with the negative moment distribution. Although this distribution is adequate for fixed support edges it will not provide for the common case of a slab simply supported on a wall. Equilibrium of

the segment implies that if the negative moments along the support edges are equal to zero there are significantly high twisting moments present which have x and y-components along the support edges. For a simple support condition, based on elastic moment fields for adjacent edge supported segments with aspect ratios of 1.0, it has been found that a top mat of steel must be provided, parallel to the support edges, to resist negative moments in the order of 30% of M^o. The distribution of these moments is such that α and ϵ and $M_{1-\alpha}/M_{\alpha}$ and $M_{1-\varepsilon}/M_{\varepsilon}$ are equal to 0.5. This is shown for a typical square segment in Fig. 3.13.

These distributions have been analytically tested under limited variable conditions so have some limitations placed upon them. First of all the elastic analyses were performed for a segment with an aspect ratio of 1.0, therefore it is recommended that caution be used when applying this segment to the SDM for other values. For a segment with adjacent edges fixed the analyses indicated that for ψ values lower than 0.7 the presence of twisting moments in the support region became a factor. Unless a top mat is placed at the corner region a minimum ψ of 0.75 is suggested for design purposes.

Under the limitations imposed on the segment, two typical moment fields were found, one for adjacent fixed edges and the other for simply supported adjacent edges. The basis for these moment fields was elastic solutions found for typical adjacent edge segments, 3.0×3.0 m square, under

a uniform load of 12.0 kN/m². The x-moment contour plots are shown in Figs. 3.14a and 3.14b with the former correspnding to the fixed case and the latter the simply supported case.

In selecting reinforcing for the adjacent edge supported segment there are two cases to consider. First for fixed edge supports a uniform top mat is recommended along the full length of the support edges. Secondly, for simply supported edges it is recommended that two thirds of the steel be concentrated over half the segment length from the supported corner, with the remaining third placed along the edge lengths beyond the half-way point. This is consistent with the edge moment distribution shown in Fig. 3.13. For both support cases the positive steel is uniformly distributed throughout the segment.

For the slab shown in Fig. 3.15, where the supported edges of the segment are discontinuous, additional negative moment resisting steel is required along the span edges to resist the support moments from adjacent segments. The lower portion of the slab in Fig. 3.15 has been roughly divided into segments with two corner supported segments, next to the adjacent edge supported elements, shown cross-hatched. From section 3.2 it is known that M^- from the cross-hatched segments is to be distributed over ϵs . This results in a negative moment over part of the span edge of the adjacent edge supported segment. To reinforce for this moment it is suggested that the moments acting on the adjacent edge segment be resisted according to the guidelines given above

and that additional top steel be provided to resist M^- and distributed over ϵs where ϵ is equal to 0.4. A similar procedure is followed for interior partitions and the same recommendations hold for y-direction moments.

The case of a segment with adjacent edges of different support conditions was not investigated and the use of such a segment is not recommended without further study.

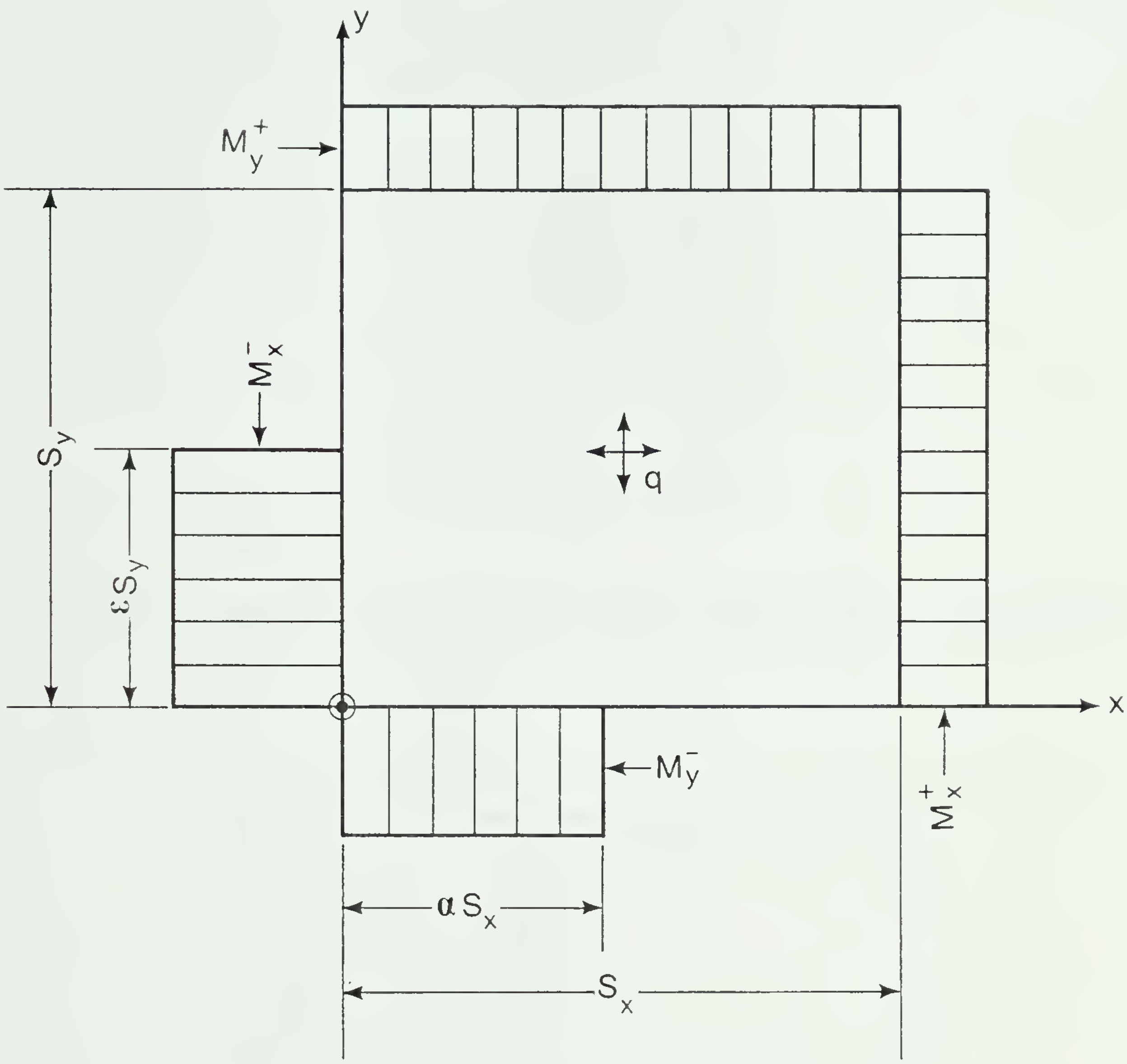
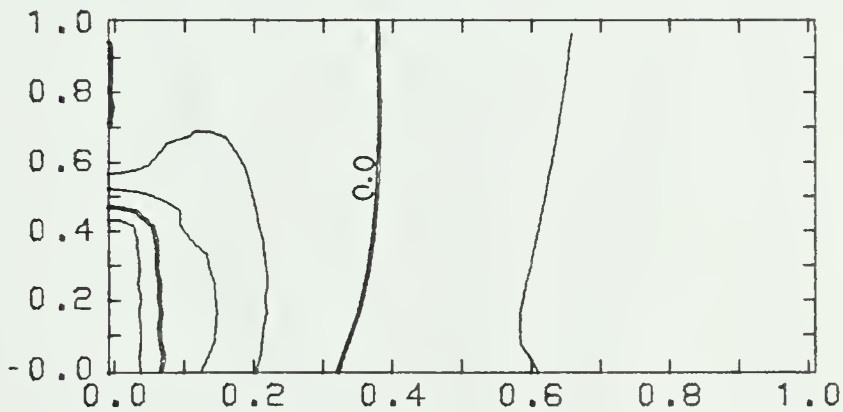


FIGURE 3.1 Corner Supported Segment with Negative Moment Concentrated Over the Support

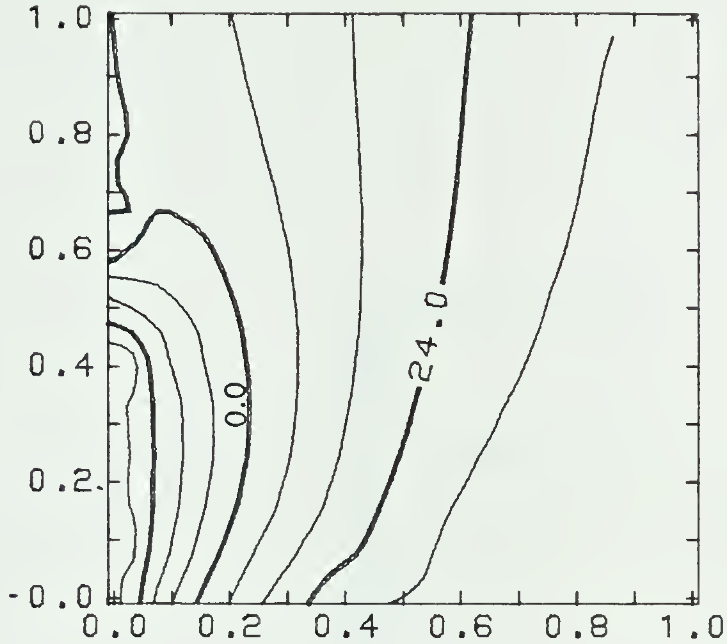
a. Corner Supported Segment with Asp. Ratio Equal to 0.5



b. Corner Supported Segment with Asp. Ratio Equal to 2.0

FIGURE 3.2 Position of Zero Moment Line in Corner Supported Segments with Different Aspect Ratios

ψ=0.5



ψ=2.0

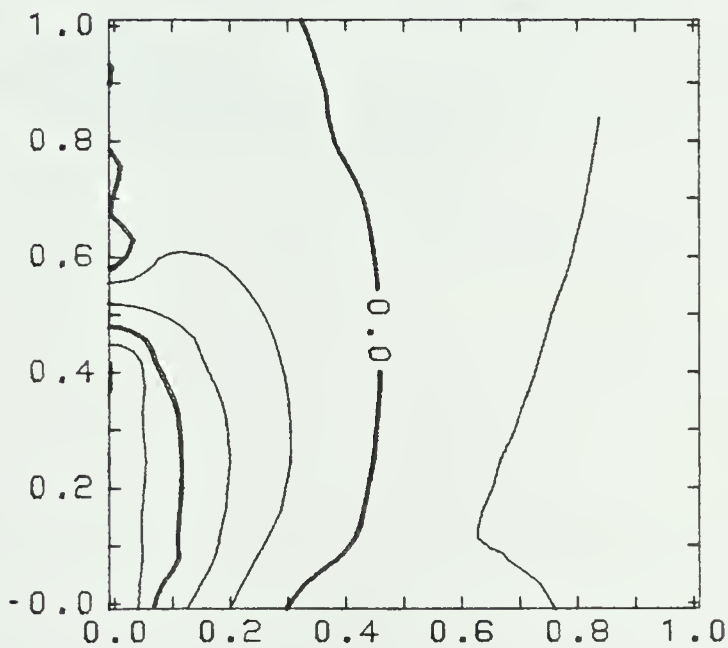


FIGURE 3.3 Position of Zero Moment Line in Corner Supported Segments with Different ψ Values

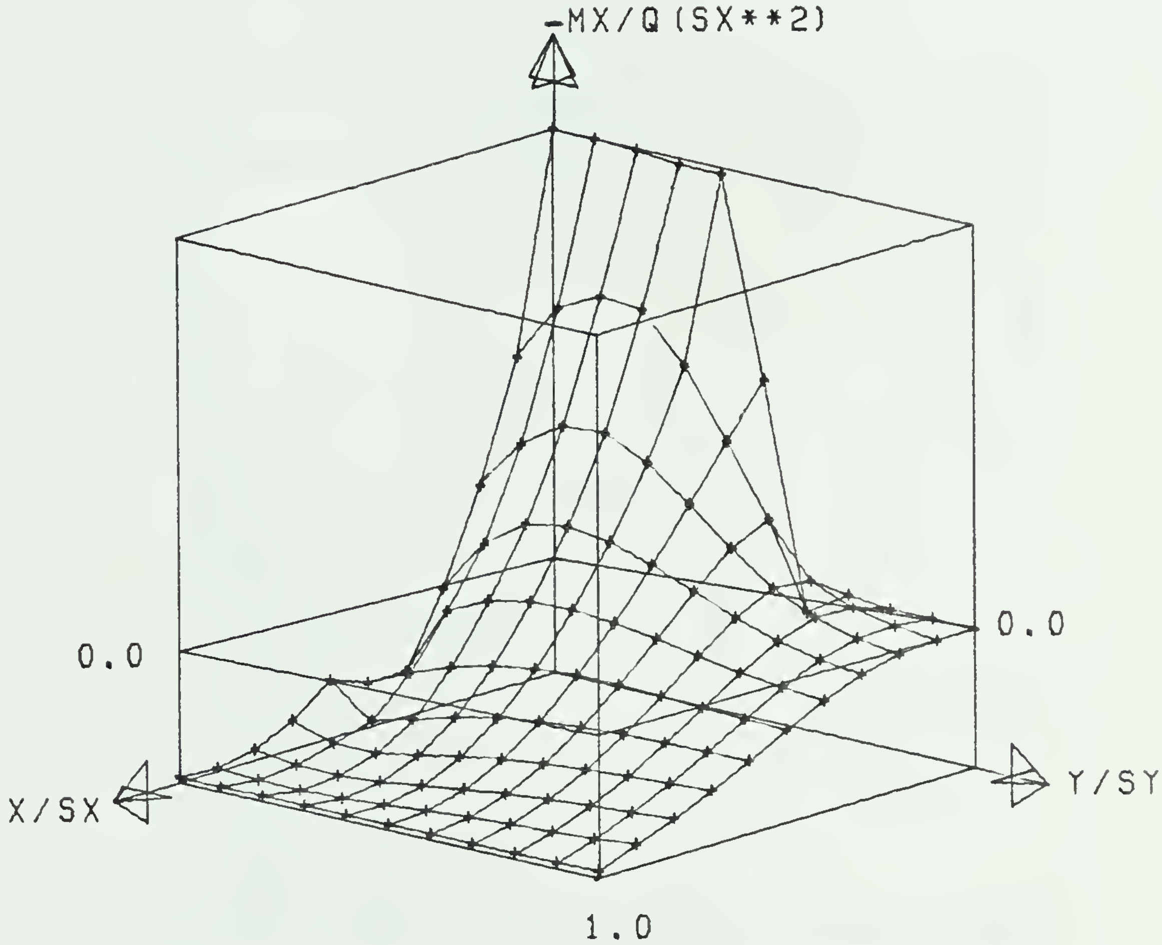


FIGURE 3.4 Profile of Moments in the X-Direction for Corner Supported Segment

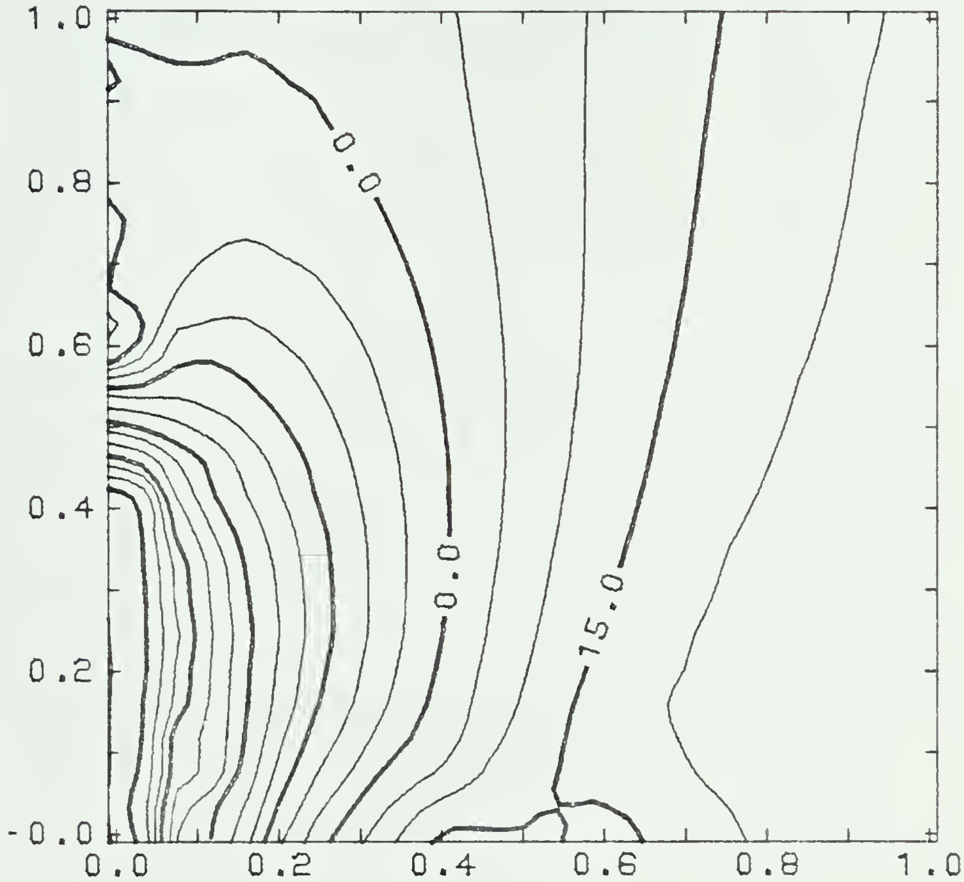


FIGURE 3.5 Contour Map of X-Direction Moments(kN-m/m) for Corner Supported Segment

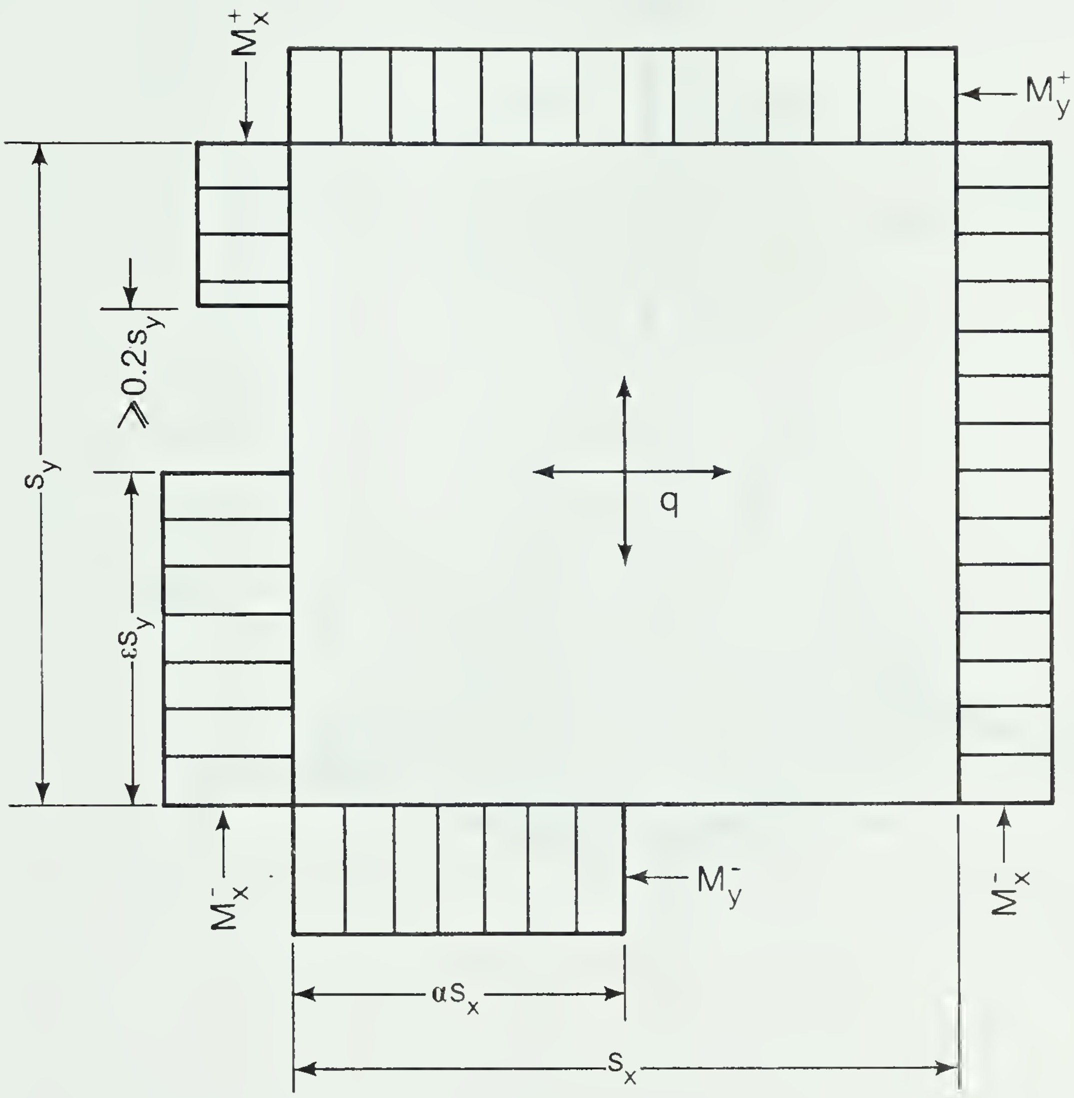


FIGURE 3.6 Plan View of Corner Supported Segment With Edge Moments of Opposite Sign

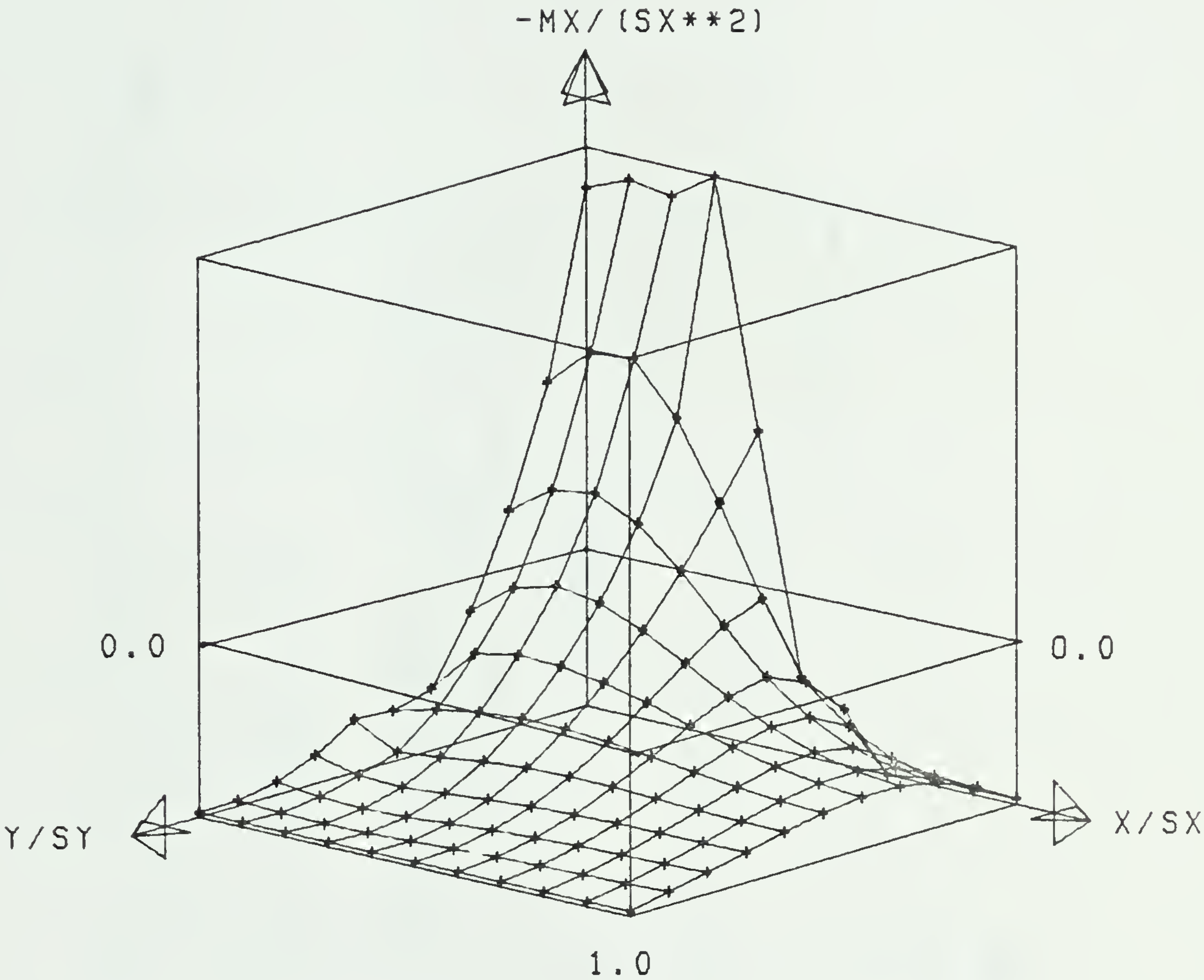


FIGURE 3.7 Profile of X-Moment Field from Corner Supported Segment with Opposite Moments along Support Boundary

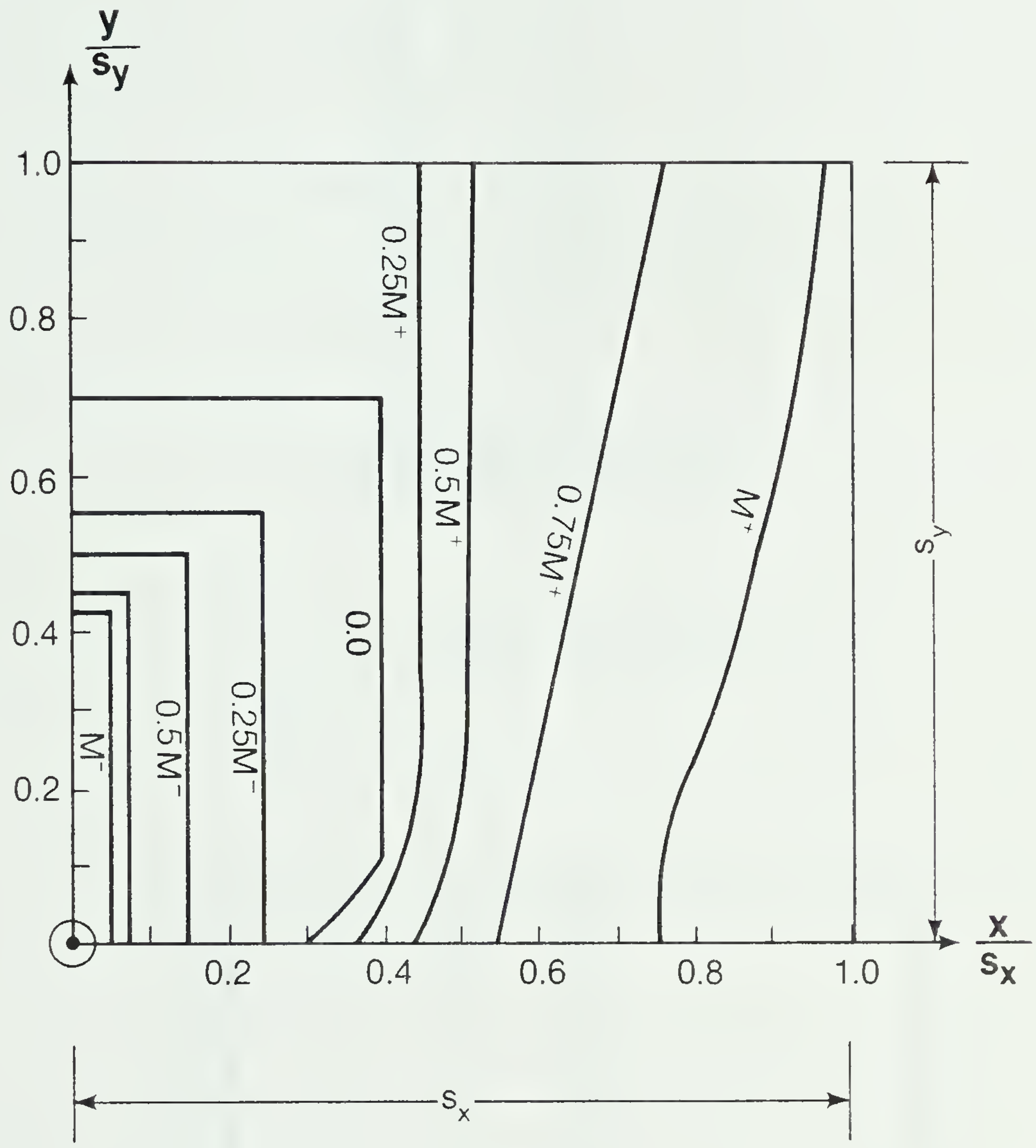
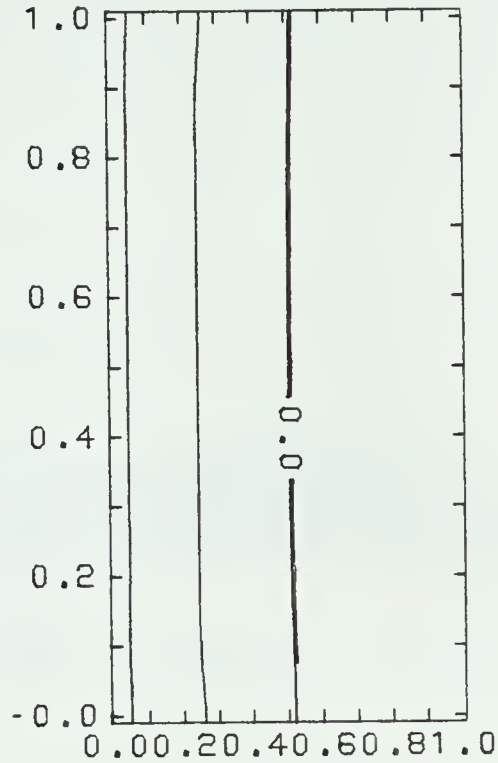
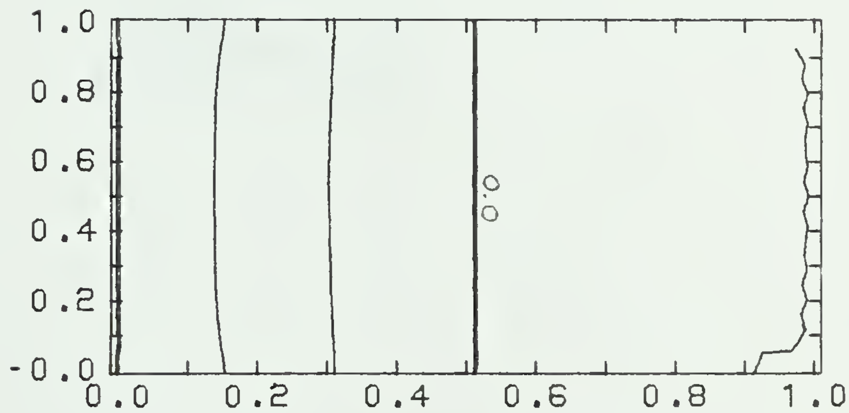


FIGURE 3.8 Simplified Moment Field for Corner Supported Segment with ψ=1.5 and ϵ=0.5



a. Aspect Ratio=0.5



b. Aspect Ratio=2.0

FIGURE 3.9 Edge Supported Segments with Varying Asp. Ratios

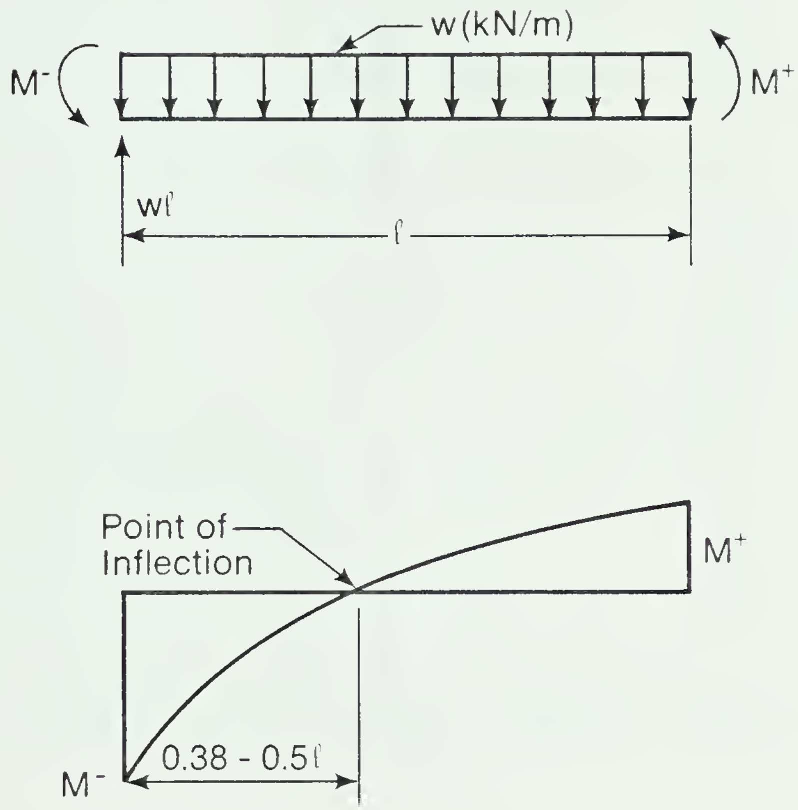


FIGURE 3.10 Profile of Moments for an Edge Supported Segment with $\psi = 1.5$

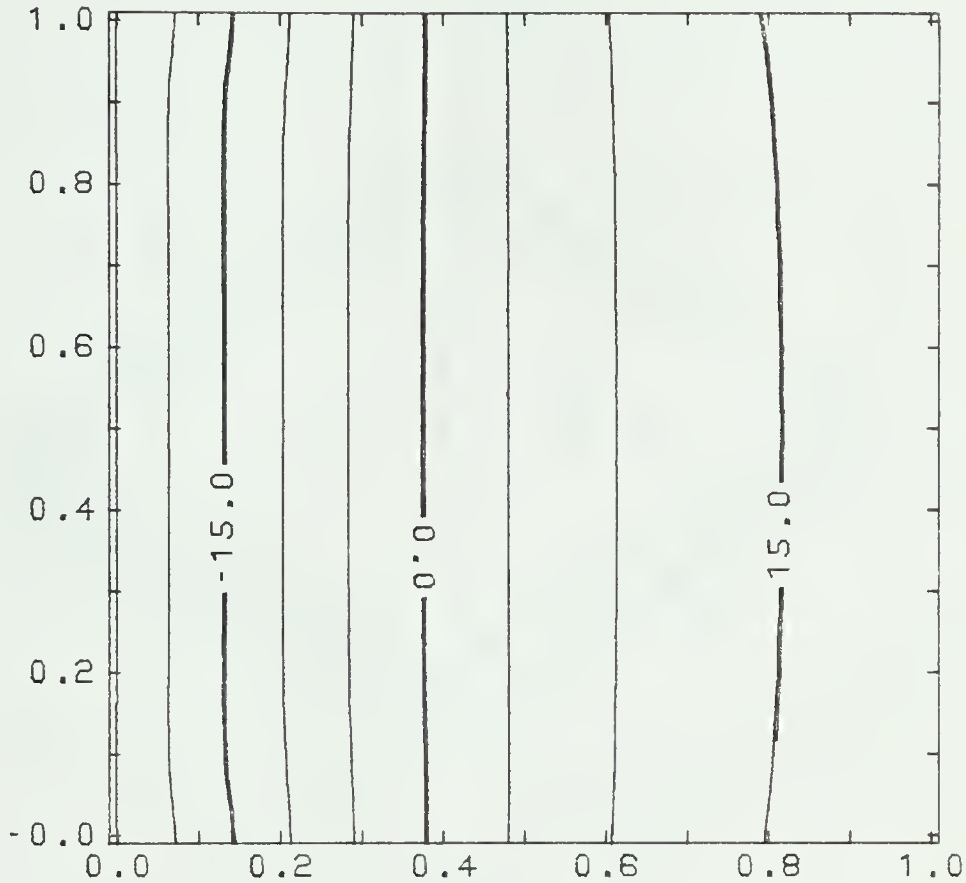


FIGURE 3.11 Contour Map of X-Direction Moments (kN-m/m) - Edge Supported Segment

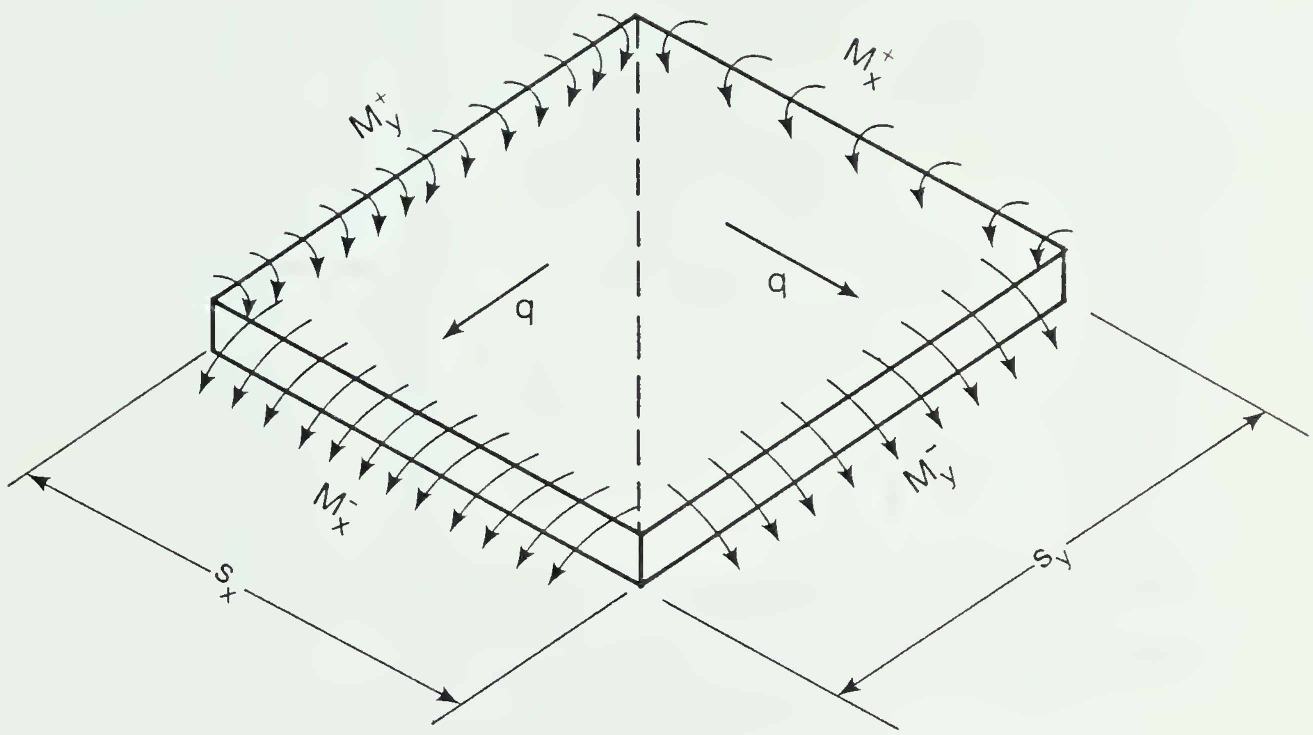


FIGURE 3.12 Distribution of Positive and Negative Moments for Adjacent Edge Supported Segment

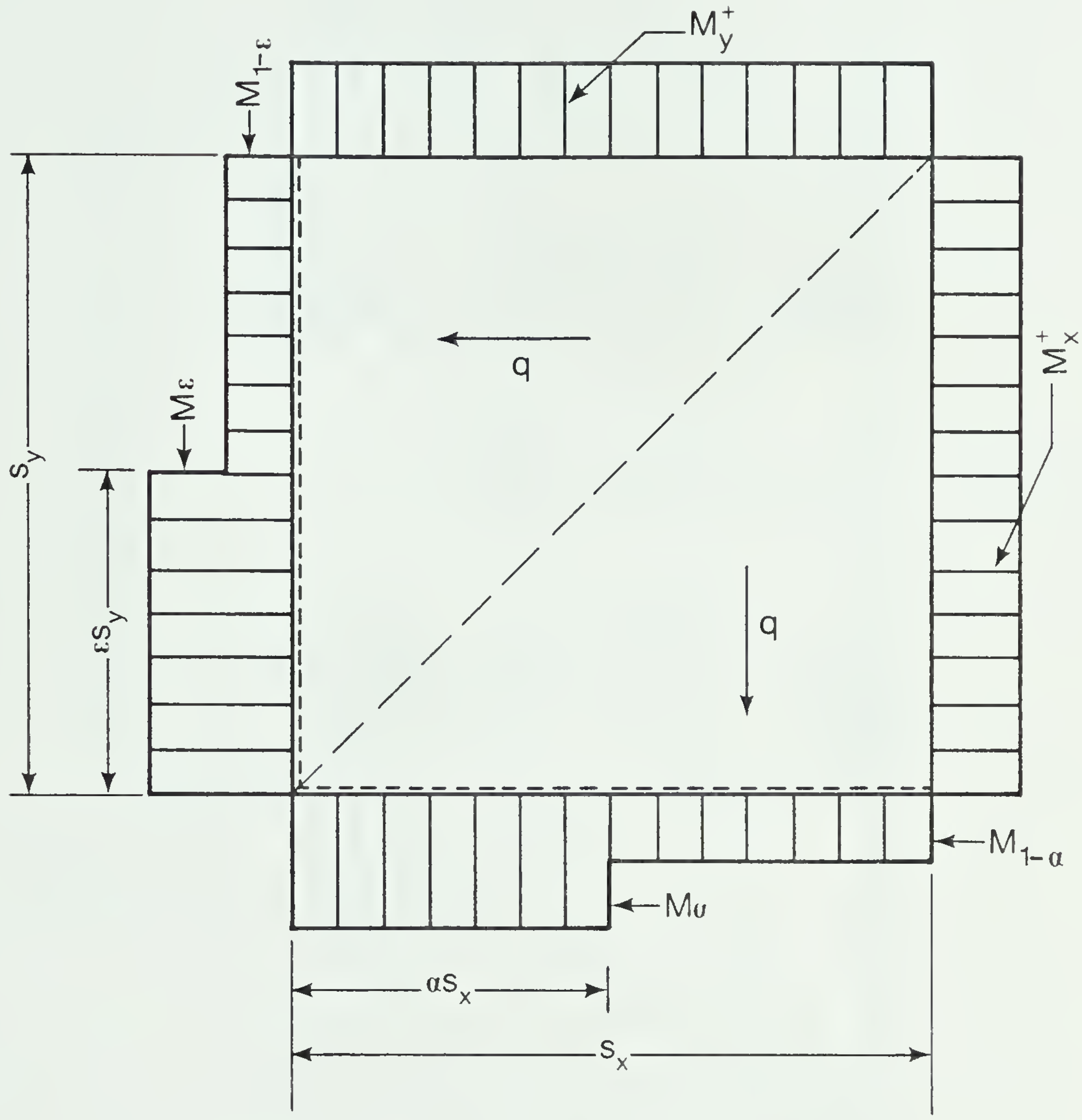
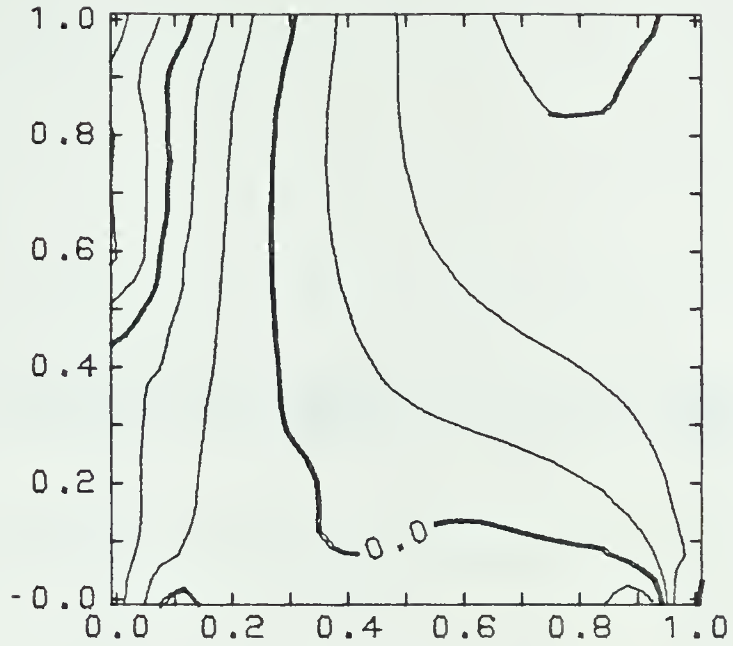
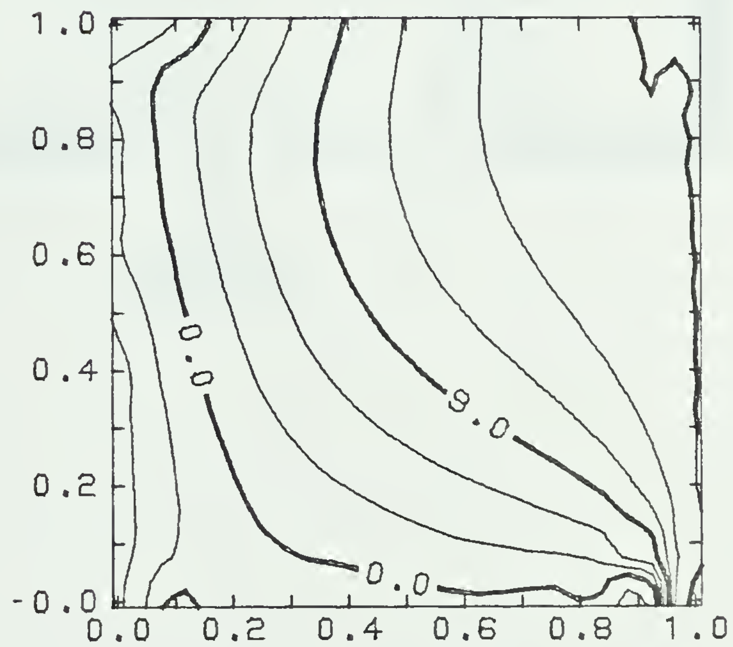


FIGURE 3.13 Distribution of Twisting Moment Components for Simply Supported Adjacent Edge Segment



a. Fixed Support Case



b. Simple Support Case

FIGURE 3.14 X-Moment Contours (kN-m/m) for Adjacent Edge Supported Segments

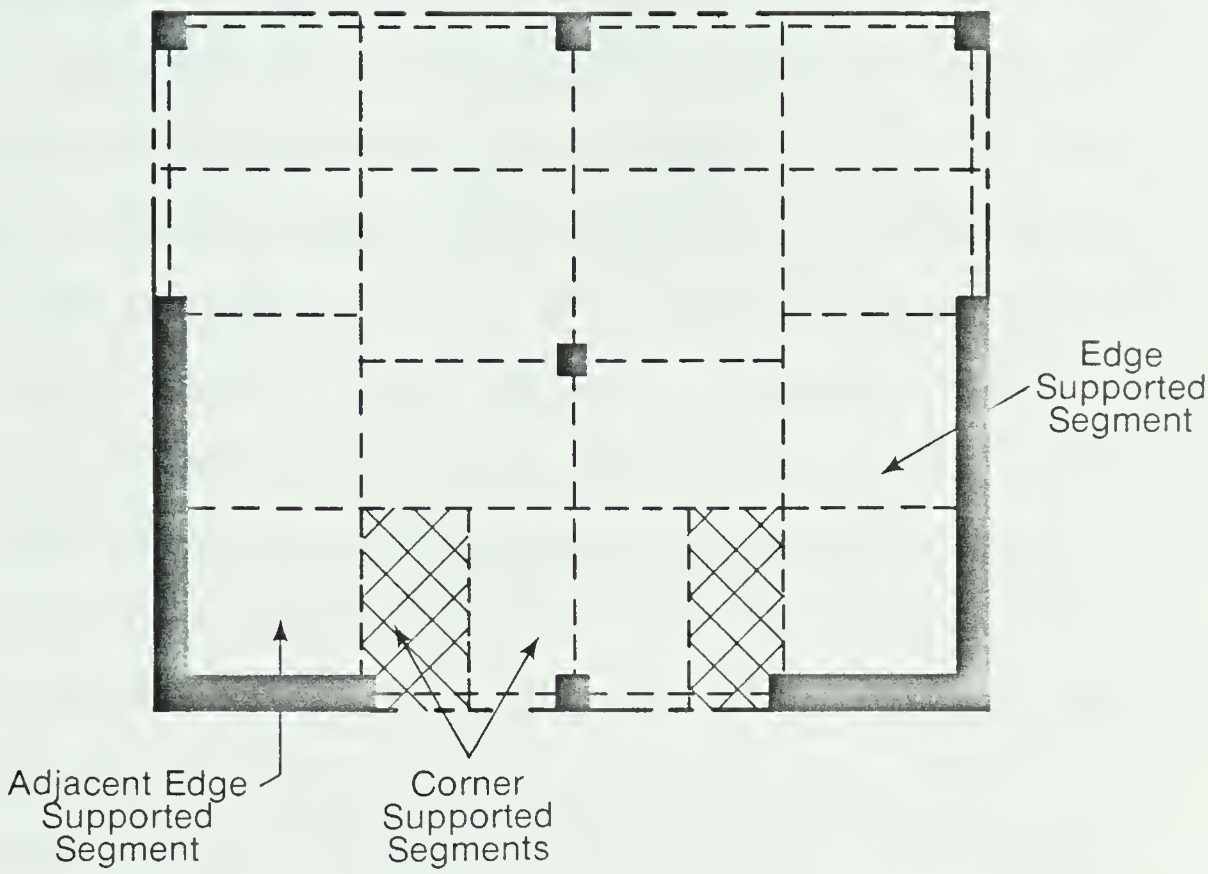


FIGURE 3.15 Slab Showing Adjacent Supported Segments with Discontinuous Edges

4. Design of Flat Plate Structures

4.1 Introduction

The segment design method was developed for easy application in the design of general flat slab problems. To demonstrate this for flat plate structures two design examples are presented in this chapter. The first example is a regular column supported slab similar to the analytical slab used in chapter 2, but with the point supports replaced by actual columns. This example is intended to introduce the design procedure and familarize the reader with some of the decisions which have to be made. The second example is a design of an irregular flat plate with a random column layout to demonstrate the scope of the procedure. Both of the designs are carried out using the step form of the procedure given in chapter 2 with a short discussion elaborating on the options available within each step. Intentional errors in selection of certain values were introduced so that the ease with which corrections or modifications of the design can be made may be illustrated.

The material properties are the same for each example and are listed below along with the unfactored live load and column dimensions.

concrete compressive strength	30 MPa
steel yield strength	400 MPa

uniform design load	3.4 kN/m^2
column sizes	400 x 400 mm

4.2 Example I - Regular Flat Plate Slab

A plan view of the slab under consideration is shown in Fig. 4.1. The geometry was chosen to have double symmetry in order to reduce the amount of necessary calculations.

4.2.1 Selection of Trial Slab Thickness

Step one in the design procedure is to select a trial estimate of the slab thickness. Based on the requirements in chapter 7 of CSA-A23.3(12) the minimum slab thickness for slabs without beams, for a typical.panel, is found from equation 4.1.

$$h=[\ell n(800+fy/1.5)]/36000 \qquad (4.1)$$

For the exterior panel in Fig. 4.1 a minimum thickness of 183 mm is calculated based on a clear span length of 5600 mm and a slab depth increase of 10% to account for the discontinuous edge. Based on this an initial value of 190 mm was chosen resulting in a factored design load of 12.0 kN/m^2.

Preliminary shear checks carried out on the slab indicate that the trial thickness of 190 mm is adequate to

prevent punching or beam shear failure.

4.2.2 Selection of Segment Boundary Lines

Because of the double symmetry in this problem, only one quarter of the slab must be considered. The boundaries of this quarter section also form segment boundaries, since lines of symmetry correspond to lines of zero shear.

The interior segment boundary lines are selected with the aid of the recommendations of section 2.2.2. For the exterior panels, in each direction, it is recommended that the zero shear line be located at 0.45 of the span length. This results in segment lengths of 2.90 m and 3.30 m, with the former corresponding to the outside segment and the latter the first interior one. The segments in the interior panels are bounded by lines of symmetry, as previously mentioned, and have lengths of 3.0 m.

The division of the slab quarter into the individual segments, in the x-direction, is shown in plan in Fig. 4.2a. Fig. 4.2b illustrates the segment dimensions in the y-direction. Note that when calculating total segment moments, the clear span is to be used in the direction under consideration and the center-to-center span in the transverse direction.

4.2.3 Calculation of Total Segment Moments

The total segment moments in the x-direction for each of the segments, is found from equation 2.1 where s_x is the clear span in the x-direction and s_y is the full segment length in the y-direction.

$$M^o{}_x = 1/2 q s_x{}^2 s_y \qquad (2.1)$$

These moments are given in brackets for the individual segments in Fig. 4.2. Because of the symmetry involved the y-direction moments are equal in magnitude and will be not discussed in this example.

For an office design situation it is recommended that a record of all the moment calculations be kept on large plan drawings of the slab. This makes it easier for the designer to keep track of his calculations and compare moment fields as well as facilitating the ease in which corrections may be made. This procedure is demonstrated in this example.

4.2.4 Splitting of Segment Moments into Negative and Positive Components

The initial splitting of segment moments into negative and positive components is based on choosing a uniform positive moment field corresponding to the capacity of shrinkage and temperature reinforcement and on selected ϕ factors. If the positive and negative moments are selected based on ϕ values alone, depending on the value of ϕ chosen, the designer may or may not come up with a suitable

distribution of moments. A poor choice of ψ may lead to moment unbalances at the supports which are too large to be carried by the columns or non-compatable moments at adjacent segment boundaries.

In this example an initial distribution of the static moment will be made by choosing values of ψ to demonstrate the problems with this approach and show the ease with which corrections can be made. The second trial is based on the recommended procedure of starting with a uniform positive moment intensity. The initial values of ψ selected, which are reasonable but not good choices, are as follows:

type of segment	ψ
exterior	0.80
first interior	2.00
other interior	1.75

Note that the ψ value of 0.8 chosen for the exterior segments is larger than that recommended in chapter 2.

With the chosen ψ values the positive and negative moments for each segment are found using equations 4.2 and 2.6.

$$M^+_x = M^o_x / (1+\psi) \qquad (4.2)$$

These moments are shown in Fig. 4.3 where the negative

moments, M^- are shown on the support boundary lines and the positive moments, M^+ shown on the span boundary lines. All moments are expressed in units of kN-m.

There are two major points to note about the moments shown in Fig. 4.3. Firstly, it is evident that there is no continuity of moments along the positive moment boundary line in the exterior panel. This violates the first rule of lower bound theorem which requires that equilibrium must be satisfied at all points. The second point of interest is that there is a high percentage of moment in the exterior span given to the outside columns. Unless the exterior columns are to be extremely stiff for reasons other than supporting the slab it is not economical or desirable to have such high moments. It is evident that a smaller value of α should have been chosen for the exterior segments.

For the second trial a uniform positive moment is selected which must be at least as great as that required for shrinkage and temperature requirements. In CSA-A23.3(12) this is given as:

$$As = \rho \times b \times h$$

where $\rho = 0.0018$ for $fy \geq 400MPa$ and 0.0020 for $fy \leq 400MPa$.

For this example, with h equal to 190 mm the required minimum steel area is 342 mm^2/m, requiring 10 M bars at 292 mm. Although the live loading is not large a positive moment somewhat greater than the minimum is selected, say 10 M bars at 250 mm giving a moment capacity of 22.6 kN-m/m.

Using the 22.6 kN-m/m as the positive design moment the negative moments for each segment were calculated from equation 2.6 and are shown in Fig. 4.4 by the second row of numbers along each support line. To better illustrate the 'note keeping' part of the procedure the positive and negative moments from the initial trial are also shown in the first row of numbers.

To ensure that those moments will provide a serviceable slab the ψ values were checked for each row of segments. In the exterior panel the smaller segments had ψ values of 0.66 and the larger segments ψ equal to 1.55. The segments in the interior panel had ψ equal to 1.08.

Based on the recommended values of ψ in Fig. 2.7 and chapter 2 some changes were deemed necessary. In the exterior panel the positive moment was increased to 25.3 kN-m/m which lowered ψ in the smaller exterior segments to a more economical value of 0.48. In the interior panel it was found that ψ was too low. If the positive moment is not decreased the negative moment in each interior segment must be increased. In the example ψ was increased to 1.60. These changes are shown in Fig. 4.4 by the third row of numbers.

4.2.5 Distribution of Edge Moments and Comparison with an Elastic Moment Field

In Fig. 4.4 two distinct lines of positive moment are evident. Since these moments are to be distributed uniformly through the segments two positive moment fields result, one

for the exterior panels and a second one for the interior panels. The respective magnitudes of these moments are 25.3 kN-m/m for the exterior panels and 22.6 kN-m/m for the interior panels. Reinforcement for these moments will be placed in a uniform mat extending throughout the slab.

The negative design moments per unit width of slab are found by by distributing the support moments in Fig. 4.4 uniformly across ϵs_y, where for this example ϵ equals 0.5. The value of 0.5 was selected from Fig. 3.8 but it would not be overly conservative to distribute the negative moments with ϵ equal to 0.55 or 0.60. It is a design decision which is left up to the designer. The unit moment resistance at the support of a segment may be found from equation 4.3 where m^-_x represents the moment resistance in kN-m/m.

$$m^-_x = M^-_x / \epsilon s_y \tag{4.3}$$

The design moments are shown on a plan of the slab as illustrated by Fig. 4.5. The numbers on top of the arrowed lines represent the design moments in kN-m/m and the average elastic moments, shown in square brackets. The number below the lines indicates the distance ϵs_y, in meters, over which the design moments are to be distributed. The elastic moments were obtained from an elastic solution of the slab using *Hybslab*(11).

Comparing the two sets of moments given in Fig. 4.5 it can be seen that the total support and total span moments

from the Segment Method are within 10% of those from an elastic solution.

The distribution of moments is also quite similar, the only difference being that the Segment Method has no negative moments in the 'middle strip' as defined in CSA-A23.3(12). For regular slabs CSA-A23.3(12) defines column and middle strip regions which are continuous across the slab. Both positive and negative moments are distributed laterally across these strips based on certain guidelines. In the Segment Design Method a uniform distribution of positive moment is used across each segment and across the entire slab if possible. The negative moments are restricted to top mats defined by lengths of segments supported by the column. Strip definitions are not used, however the top mats in the SDM compare roughly to the intersection of two column strips as defined by reference 12.

For irregular slabs the code definitions of strips have no meaning and the previous guidelines for distributing moments do not apply. In the Segment Design Method the regions over which the moments are distributed do not depend on column and middle strip definitions and are defined for any column layout.

The design moment distribution calculated from the SDM, having met the requirements of a lower bound solution and approximating an elastic solution may be considered as a moment field which will provide a safe and serviceable slab.

4.2.6 Selection of Reinforcement

The selection of reinforcing steel for the slab is straightforward and with the aid of the tables in Appendix A, very simple. The minimum steel requirements in the interior panel have already been found and a bottom mat of No. 10 M bars at 250 mm centers provided. For the positive moment in the exterior panel of 25.3 kN-m/m a bar spacing, assuming No. 10 M bars, of 200 mm was selected from Table A.1, providing a capacity of 28.1 kN-m/m.

In determining the steel requirements for the top mats each column line is considered in turn. For the exterior column line a moment resistance of 24.4 kN-m/m must be provided. This is accomplished with the use of No. 10 M bars at 200 mm centers. The resulting resistance is 27.9 kN-m/m. Slab moments at the interior columns are significantly higher, therefore Table A.2 will be used. For a moment of 64.7 kN-m/m No. 15 M bars are provided, spaced at 150 mm giving a capacity of 65.0 kN-m/m. The reinforcing layout for the slab is shown in plan in Fig. 4.6.

In detailing the steel layouts, the first thing looked at was the inflection points for the negative moment. With reference to Fig. 3.8, it is seen that for a corner supported segment with ϵ equal to 0.5, the point of inflection occurs at 0.4 of the segment length from the column. It should also be noted that the maximum moment may be reduced to a half at 0.20 of the segment length. All other detailing requirements are in accordance with the

appropriate clauses in CSA-A23.3(12), including the calculation of the embedment length for the steel which must be added on to the theoretical cutoff points in Fig. 3.8.

For the positive steel the development lengths were calculated with regards to clause 11.6.6 of the concrete code(12). All bars perpendicular to a discontinuous edge must be adequately anchored. At the interior columns 50% of the bars must be within 75 mm of the column centerline and the remainder within 0.125 of the span length unless a detailed moment-shear analysis is made.

The detailing of the top mats of steel used the moment field of Fig. 3.8 where for ψ between 0.5 and 2.0 one half of the steel may be terminated at $(0.20s+\ell d)$ and the inflection point taken at $(0.40s+\ell d)$. The embedment length for the top steel is in accordance with clause 10.4.3 of the code(12). Steel details are shown in Fig. 4.9 for a typical column and middle span section.

For comparative purposes Table 4.1 gives the minimum lengths of reinforcement, using the preset moment field and the recommendations in the code (Table 11.1)(12).

4.2.7 Evaluation of Shear-Moment Interaction

Any moments acting at the column faces must be checked to ensure that the column can take the unbalance. Since it is not a principal part of this thesis no calculations are shown. The shear-moment interaction at the columns may be checked using the requirements in chapter 11 of

4.3 Example II - Irregular Flat Plate Slab

4.3.1 Introduction

Example II was specifically selected as a design problem outside of the requirements of the code. It is too irregular for either the Direct Design Method or Equivalent Frame Method and would be time consuming and costly to provide an elastic solution in practice. It is intended to show the versatility of the Segment Design Method when applied to flat slab structures. A plan view of the slab system under consideration is shown in Fig. 4.8.

As in Example I, the first step in the procedure is to determine a trial thickness for the slab. Since there are no clear cut areas from which to select a value of the clear span an approximation was found by considering quadrilateral regions of the slab bounded by four columns. The distance between each of the columns in a particular quadrilateral was measured and the largest of these was taken as the governing span. Fig. 4.9 shows two such areas, the first being an outside quadrilateral bounded by the four columns in the top left section of the slab and the second an inside quadrilateral bounded by the innermost columns. Taking the governing center-to-center span as 6.3 m and reducing it by the column dimension results in an initial slab depth, from equation 4.1, of 175 mm. This has been increased by 10% because of the discontinuous edge, resulting in a slab thickness of 195 mm. Since this is close to the 190 mm used

in Example I the uniform factored load of 12.0 kN/m^2 will remain as the design load.

Shear checks are to be carried out as required by the code(12) based on assumed tributary areas of the columns.

4.3.2 Selection of Segment Boundary Lines

Because of the symmetry about the x-axis of the slab only the top half of the slab will be considered. The x-axis itself will form one boundary line for the lower segments.

When attempting to divide a slab with an irregular column layout, the designer must rely more on his judgement in positioning the zero shear lines then when dealing with a slab with regular column lines. Here, because of the uneven spacing, the application of the suggested guidelines, (such as exterior segments take up 0.4 to 0.5 of the panel length, etc.), is not as obvious. It will also be shown that a set of segments which result in a satisfactory moment field in one direction may not give good results in the opposite direction. This situation requires two sets of segments to complete the solution, one corresponding to moments in the x-direction and the second corresponding to moments in the y-direction. Several trials may be required before a satisfactory set of segments is found. Fig. 4.10 illustrates one possible division of the slab into segments. Because of the complexity of the slab it was easier to show the segment boundaries through the columns, however, when calculating segment moments the clear span is used in the direction

under consideration and the full span in the transverse direction. Note also that the segments are numbered on the support boundaries and are shown circled. This will make it easier to follow the steps of the design which are discussed in the next sections.

For segment boundary lines parallel to the y-axis, exterior segments were given dimensions of 0.45 and 0.5 of the panel length for slab areas in the top left and bottom right sections of Fig. 4.10, respectively. This increase from the previous example for exterior segments was selected because of the proportionate increase in tributary area of the exterior columns in those regions. The boundary lines parallel to the y-axis in the interior region were selected based on segment dimensions of half the column spacing.

Similar reasoning was used in determining the positioning of the boundary lines parallel to the x-axis.

4.4 Determination of X-Direction Design Moments

4.4.1 Calculation of Total Segment Moments

The static moments for each segment were calculated using equation 3.1 and are shown in Fig. 4.10.

4.4.2 Splitting of Segment Moments into Negative and Positive Components

Using shrinkage and temperature reinforcement as a starting point for the positive moment resistance, for a slab thickness of 195 mm, equation 4.4 gives a minimum required steel area of 351 mm^2 per meter. This results in a uniform bottom mat of No. 10 M bars spaced at 290 mm each way and a positive moment field in the x and y-directions of 20.1 kN-m as found from Table A.1. With this capacity and equation 4.3 the negative moments in each segment can be calculated. These are shown in Fig. 4.11.

Starting with the first panel of length 5.0 m at the left side of the slab, μ values were calculated for all segments. The segments numbered 2 and 3 in the first panel have zero positive moment on the boundary since these segments are cantilevered. The negative moments are equal to M^o and cannot be changed. The other exterior segments had very low values of μ. Most of the interior segments had μ values between 1.0 and 1.2. Some segments, such as 10, 18 and 19, had positive moments exceeding the total static segment moments.

Right away it is seen that a poor choice of segment boundaries has been made. A second set of segments is shown in Fig. 4.12 with the corresponding moments, M^o given in brackets. Using the same positive moment reinforcing (ie. capacity 20.1 kN-m/m) the negative and positive moments are calculated and are shown in the first row of numbers on the

span and support boundaries respectively.

The major changes to the previous set of segments was made in the upper right portion of the slab. Here portions of segments 13 and 21 were replaced by increasing the size of segment 15. This reduced the large column moments in the lower segments and reduced the support moment in segments 21 and 22 considerably. Other changes included increasing the segment area in segments 3 and 6 and increasing the length of segment 1 so as to raise their respective ψ values.

Checking ψ values for the new segments it is found that some modifications are still necessary. The two top segments, segments 1 and 4 have ψ values of 0.25 and 0.94 respectively. To obtain more satisfactorary moments these values are increased to 0.42 and 1.45 resulting in negative moments of 20.3 kN-m and 62.4 kN-m for the exterior and interior segment. The same procedure is carried out for segments 20-23.

At the center interior column the four segments, 8,9,13 and 14, have have equal negative moments except for segment 14. By assuming that none of the positive moment on that segment will be used for strength purposes the negative moment can be increased to 38.9 kN-m. Other interior column moments are adjusted to provide negative moments resulting in ψ values of 1.50.

One other segment of note is the large rectangular segment in the top right portion of the slab (segment 15). Because of its position and size it has two opposite moments

along the support edge, a negative moment of 99.3 kN-m and a positive moment of 40.2 kN-m. To maintain equilibrium of this element the negative moment will have to be increased to 139.5 kN-m. This increases the ψ value to 1.65 which brings it within the recommended range of values. All changes to the moments are shown in Fig. 4.12 with the initial positive and negative moments crossed out.

It should be mentioned that although most of the negative segment moments were adjusted based on a ψ of around 1.50 this does not imply that it is a unique or best solution. The range of values are between 1.35 and 2.0 and the selection of a particular value for ψ is left entirely to the designer's judgement.

4.4.3 Distribution of Edge Moments

The distributed moments in the x-direction are those in Fig. 4.13, along with the moments from an elastic solution in square brackets. All positive moments along segment edges are uniformly distributed and carried through the individual segments. The negative moment per unit width, for each segment, may be found from equation 4.3. In cases where adjacent moments on the same side of a support differ by more than 10% it is suggested that the moment to be resisted be taken as the larger of the two. If the difference is less than 10%, moment redistribution will allow the designer to use the average moment acting at the column face, provided the sum of the positive and negative moments in the segments

is greater than or equal to the static segment moments. The maximum allowable moment redistribution of 10% is an arbitrary criterion and consistent with recommendations in chapter 11 of CSA-A23.3(12).

The negative moments in the x-direction are distributed for the most part with ϵ equal to 0.5. A notable exception to this is at the top column supporting segments 12 and 15. Looking first at segment 15, ϵ must be at least equal to 0.45 of the segment length. Since there is also a positive moment along the support edge a minimum distance of 0.2 s is left for the transition of negative to positive moment. This has the effect of reducing the effective positive moment in segment 13 from 40.2 kN-m to 31.0 kN-m. To maintain equilibrium the corresponding negative moment will have to be increased to 65.7 kN-m.

In segment 12 so as to ensure continuity across the adjacent support boundary, its negative moment is distributed with ϵ equal to 0.86. In all other cases ϵ is equal to 0.5.

Comparing the elastic and SDM moment fields shown in Fig. 4.13 it is evident that the SDM results in a suitable distribution of moments. The negative support moments are in reasonable agreement with the SDM, in most cases, giving moments in the order of 0.9 to 1.40 of the elastic moments. The one area where there is some difference is in the top right corner of the slab. The large negative moment of 73.8 kN-m/m at the first interior column is due to the large size

of the corresponding segment. A different positioning of boundary lines in this area could be tried to reduce this moment if it is felt necessary.

The elastic positive moments shown are much lower than those of the Segment Method due to the fact that the design positive moments in the SDM take into account the minimum steel for shrinkage and temperature and not all of this capacity is used for strength pruposes.

4.5 Determination of Y-Direction Design Moments

4.5.1 Calculation of Total Segment Moments

The initial choice of segment boundaries for moments in the y-direction is the same as the initial choice for the x-direction given in Fig. 4.10. The static moments for each segment are found from equation 2.2 and are given in Fig. 4.14.

4.5.2 Splitting of Segment Moments into Negative and Positive Components

Starting with the uniform positive moment of 20.1 kN-m/m, based on minimum reinforcement requirements, the corresponding support moments are calculated from equation 2.6 and shown in Fig. 4.15.

In examining the y-direction moments from Fig. 4.15 there seems to be only one major problem area. This occurs

in the top portion of the slab along segments 12,13,15,20 and 21. These segments tend to have large s values resulting in large positive segment moments, and hence low negative segment moments. Since this mainly affects just the exterior segments along that area it was decided not to alter the segment boundaries but to provide negative moments giving exterior μ values of 0.45 and interior values of 1.65.

Note that the two top segments, segments 1 and 4, μ values in the order of 0.76. Being exterior segments it is felt that the positive moment could be increased on the span boundaries. By adding one more bar every meter to the bottom mat in this region the positive moment capacity is increased to 26.0 kN-m/m. This decreases the μ value of the segments to 0.44.

The modifications made to the positive and negative moments are shown in Fig. 4.13 underneath the crossed-out, initial moments.

4.5.3 Distribution of Edge Moments

The distribution of moments in the y-direction is carried out in the same fashion as those in the x-direction (see section 4.4.3). The distributed moments are shown along with the elastic moments in Fig. 4.16. Again there is reasonably close agreement of the negative moments between the SDM and elastic solutions. The high positive moments resulting from the SDM procedure indicate that because of

the light load the shrinkage and temperature requirements govern the capacity of the positive steel.

Another approach to designing a slab of this type is to use simplified, segment moment fields such as shown in Fig. 3.8. With a family of such figures, for different μ values a designer would simply overlay a number of these moment fields on a plan of the slab. The resulting contours would give a complete moment field for the entire slab.

Table 4.1 Minimum Lengths of Reinforcement in Slab

Position in Slab	Preset Moment Field (Fig. 3.7)	Code (T. 11.1)
Top bars at exterior columns	50% @ 0.14 n 50% @ 0.23 n	50% @ 0.2 n 50% @ 0.3 n
Top bars at interior columns	50% @ 0.16 n 50% @ 0.28 n	50% @ 0.2 n 50% @ 0.3 n

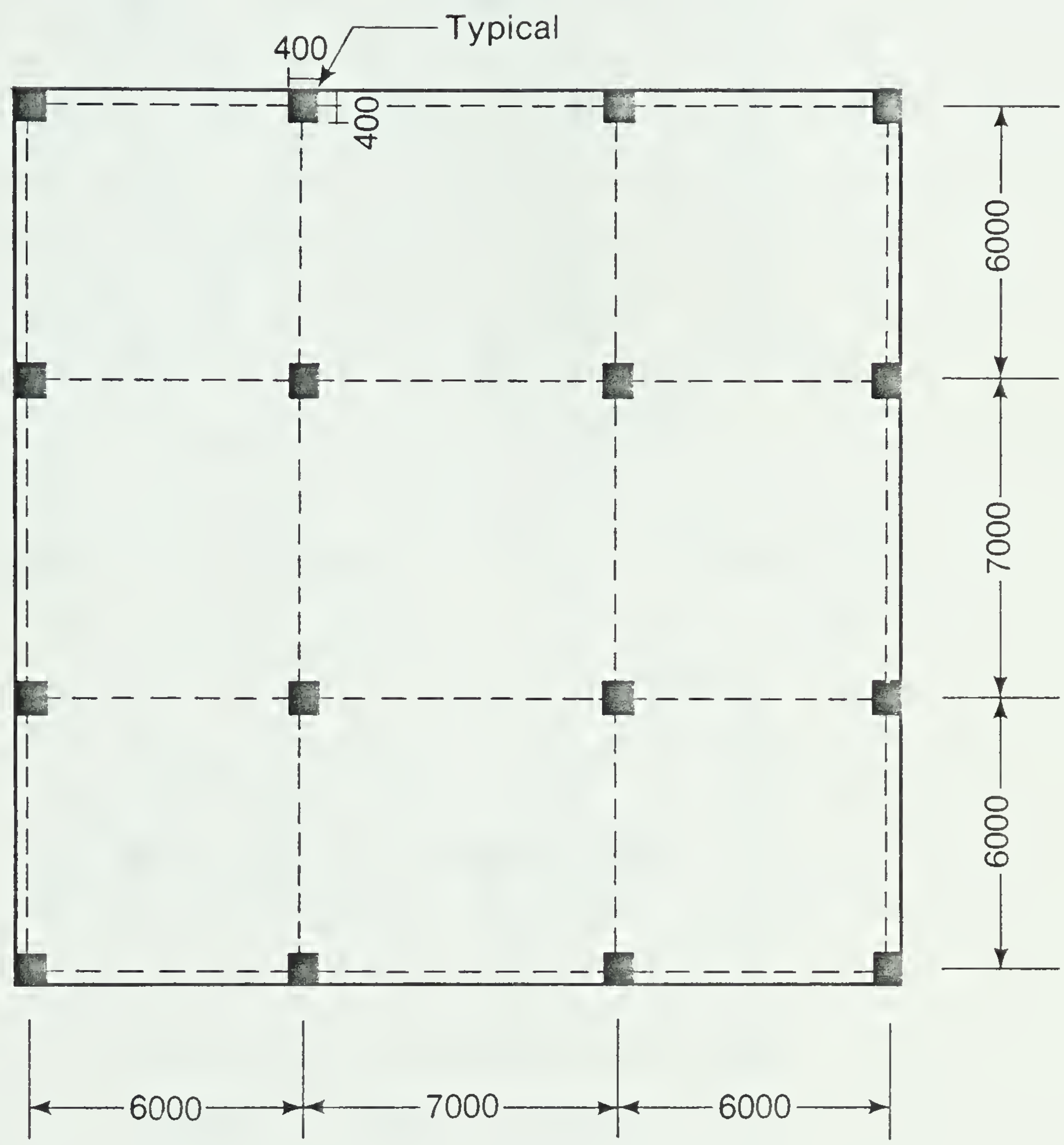
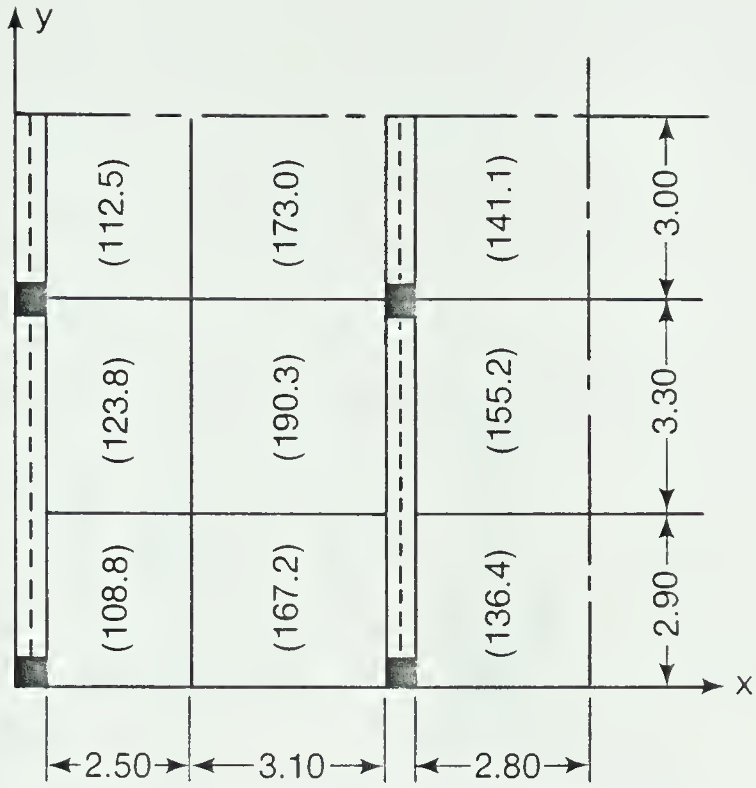
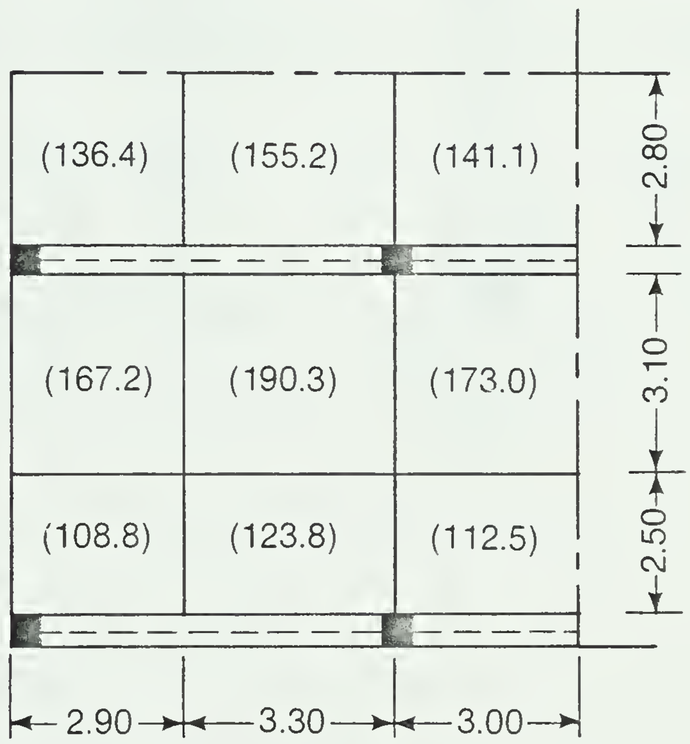


FIGURE 4.1 Plan View - Example I



a. Moments and Segments for the X-Direction



b. Moments and Segments for the Y-Direction

FIGURE 4.2 Positioning of Segment Boundary Lines and Total Segment Moments for the Slab

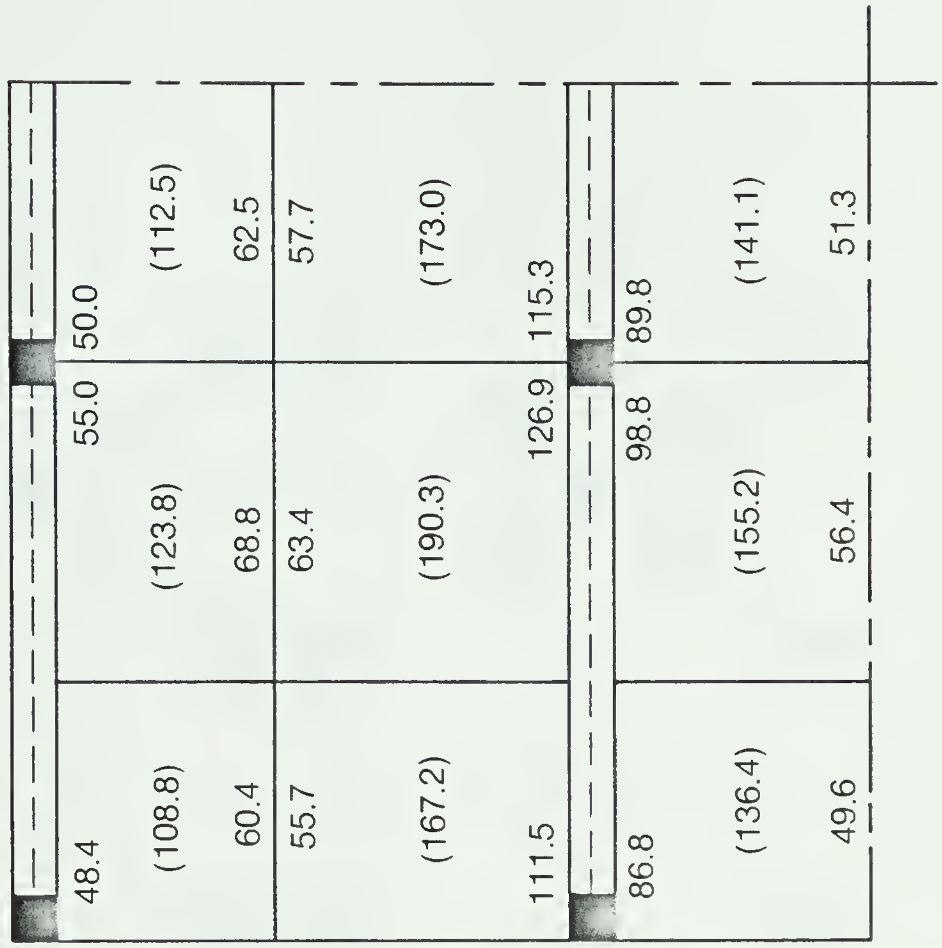


FIGURE 4.3 Initial Negative and Positive Segment Moments for Example I

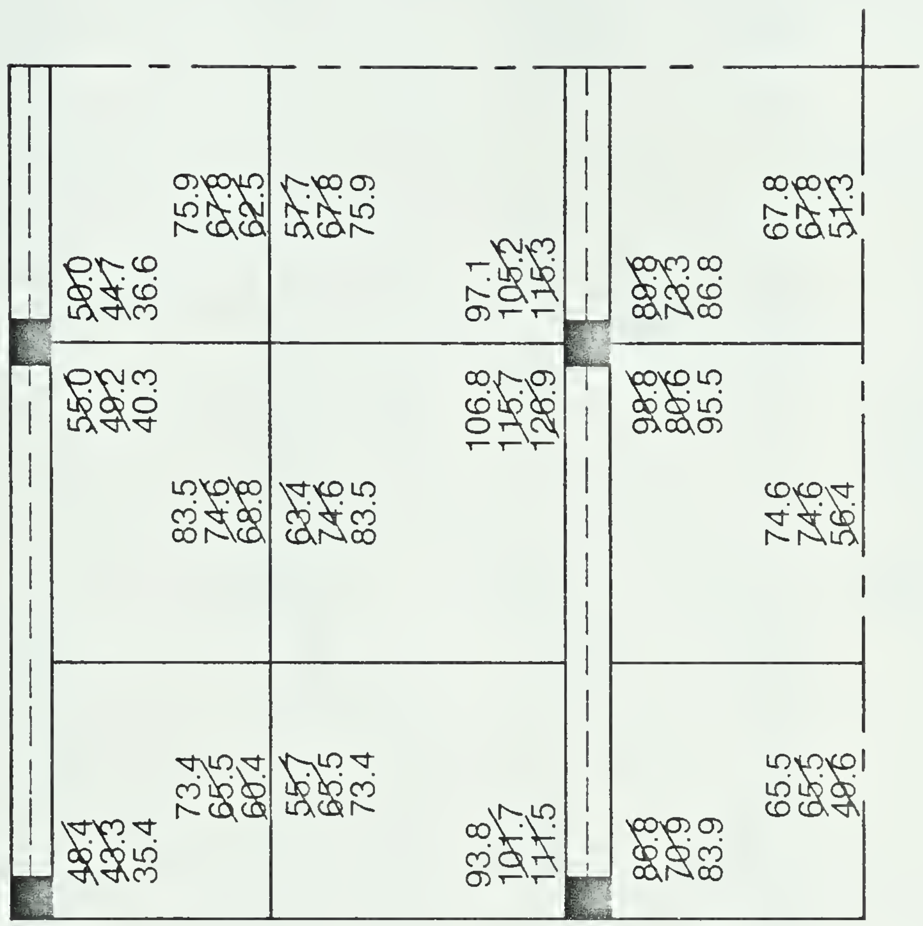


FIGURE 4.4 Final Positive and Negative Segment Moments for Example I

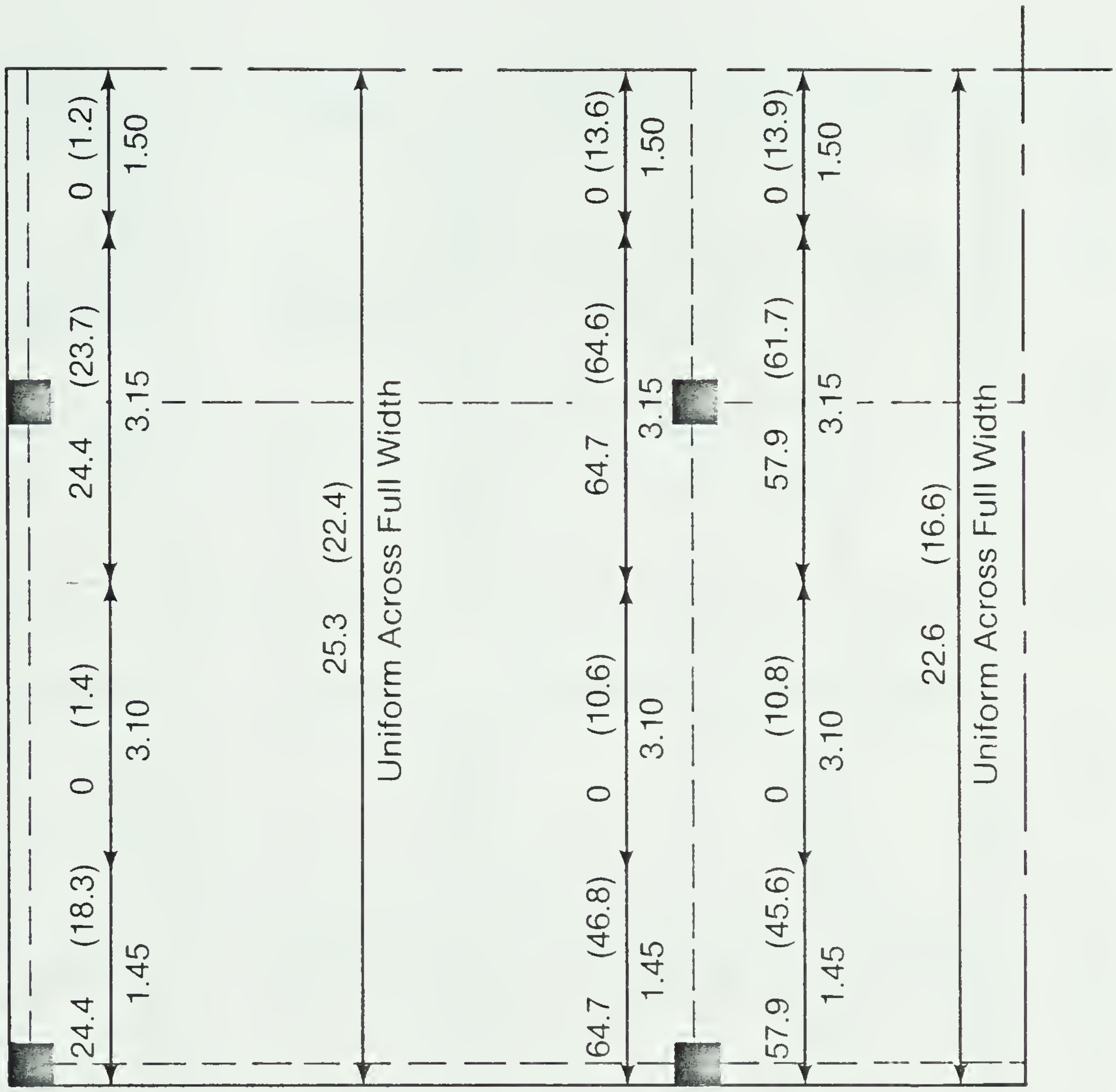


FIGURE 4.5 Distribution of Design Moments for Example I

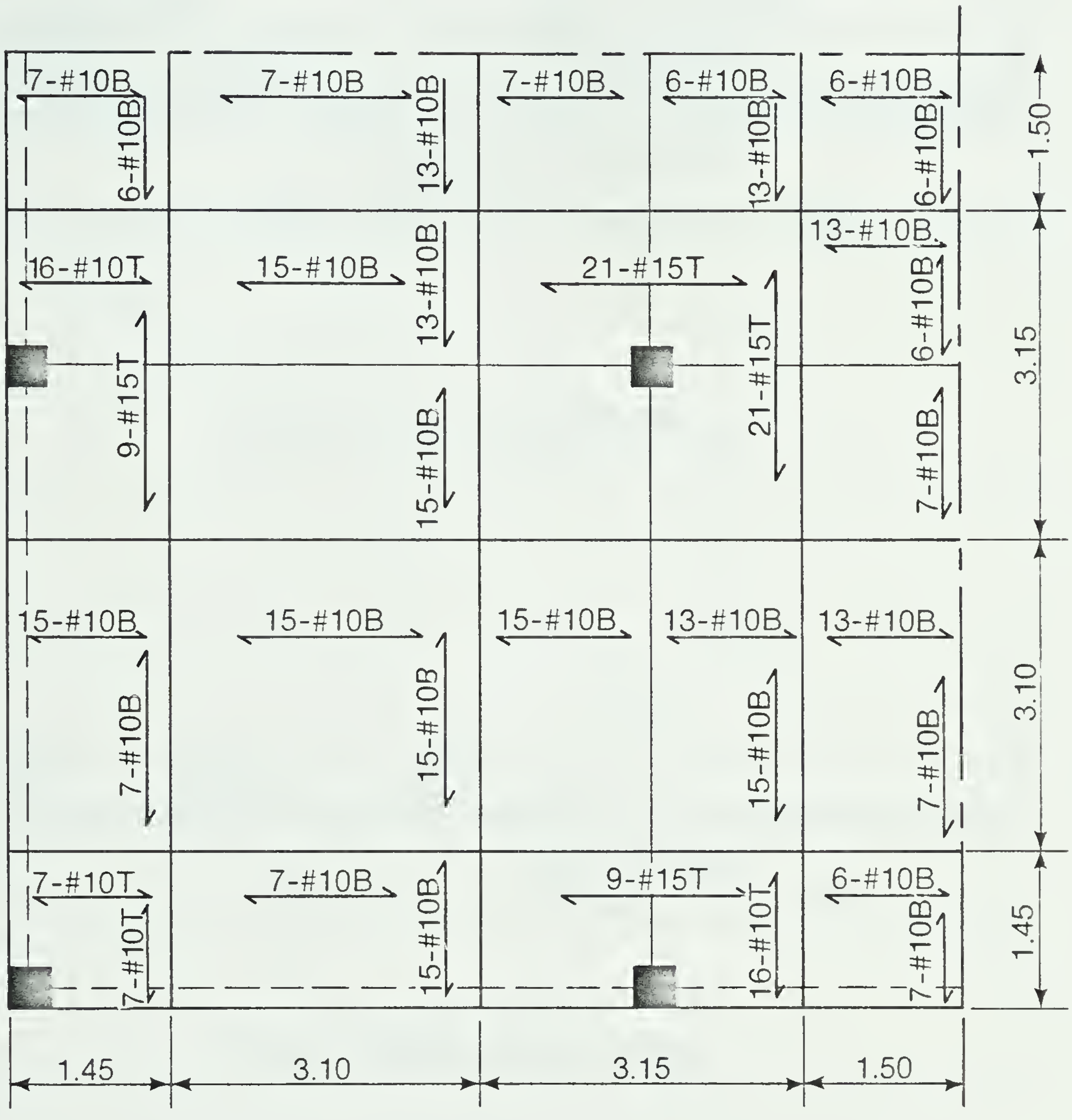
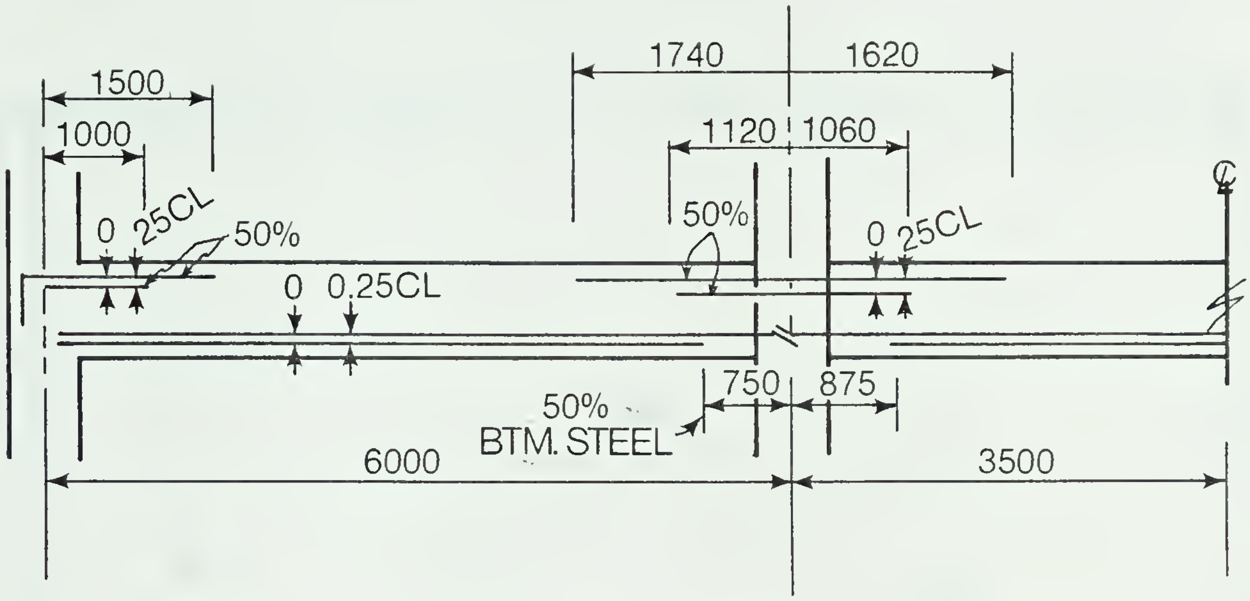
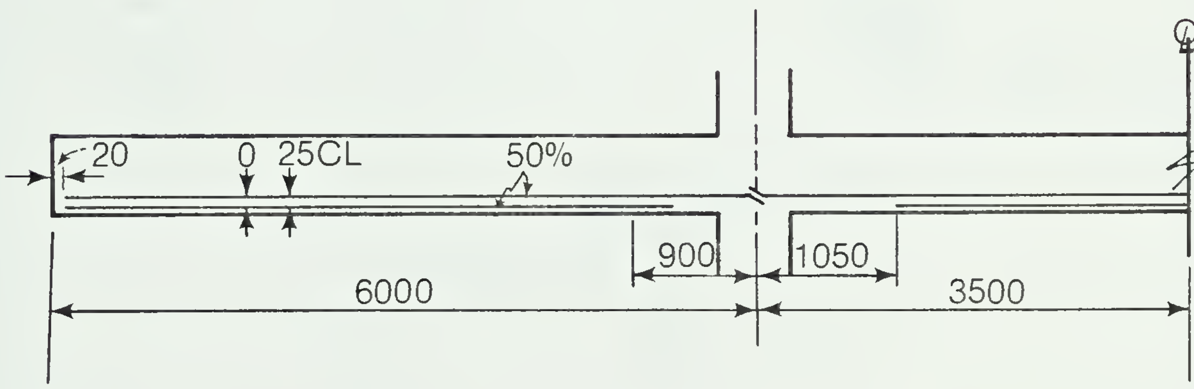


FIGURE 4.6 Layout of Reinforcing Steel



Typical Column Strip Section



Typical Middle Strip Section

FIGURE 4.7 Typical Steel Details for Slab

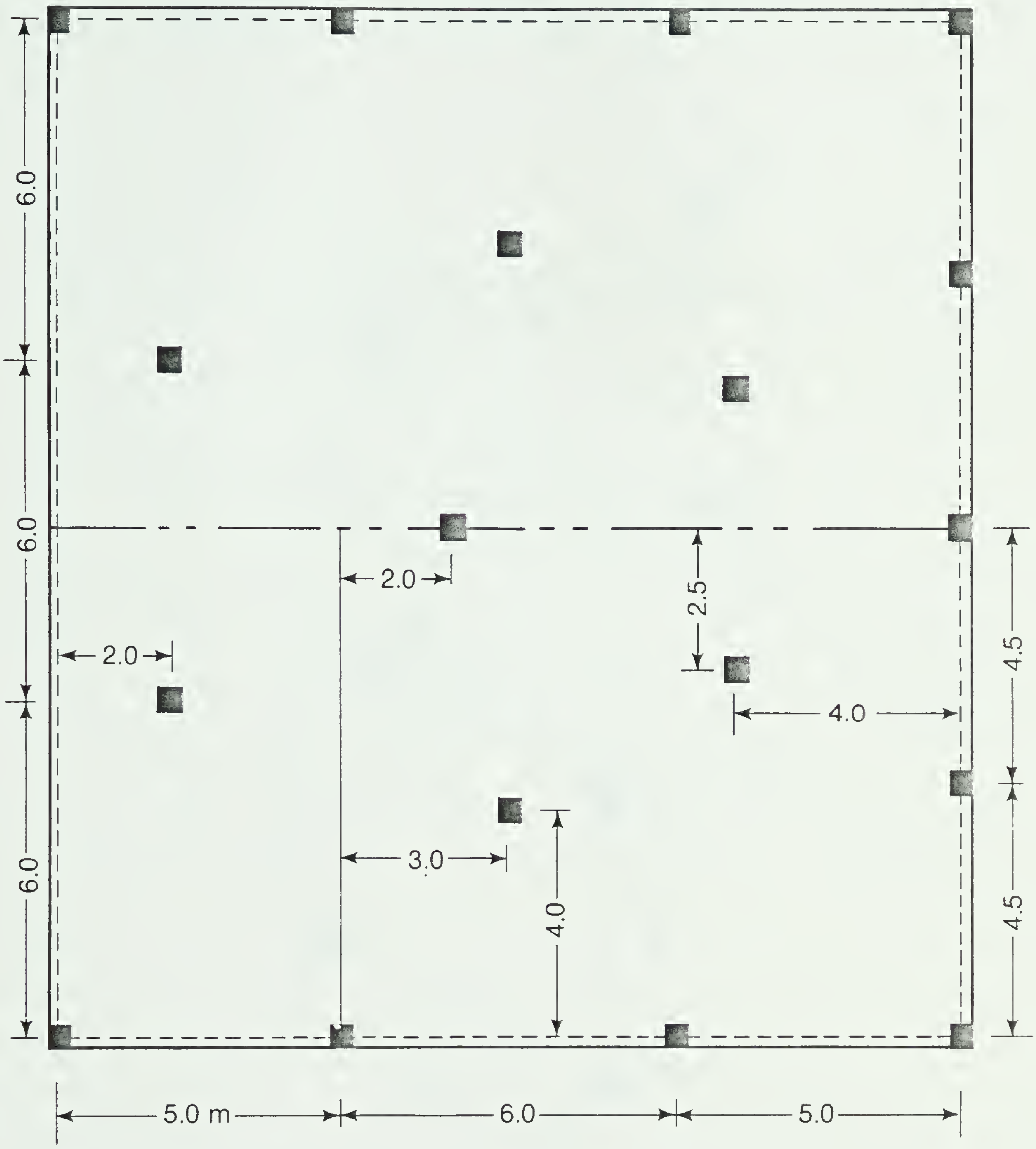
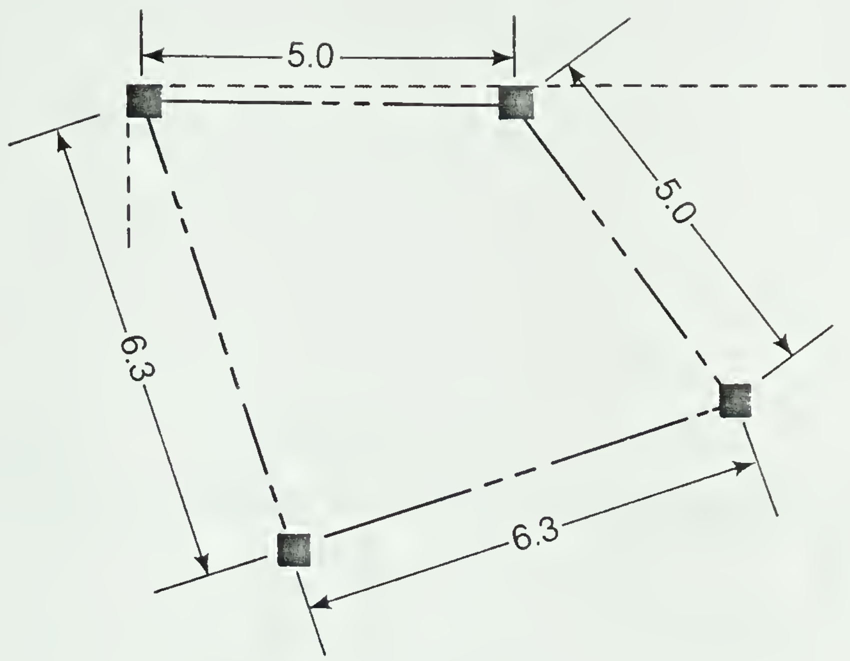
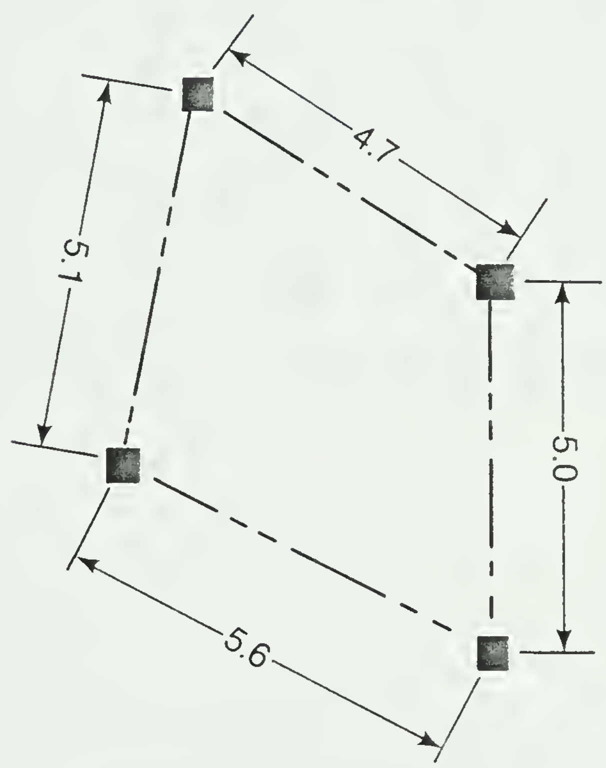


FIGURE 4.8 Plan View - Example II



a. Exterior Quadralateral Region



b. Interior Quadralateral Region

FIGURE 4.9 Quadralateral Regions Used for Determination of Slab Thickness

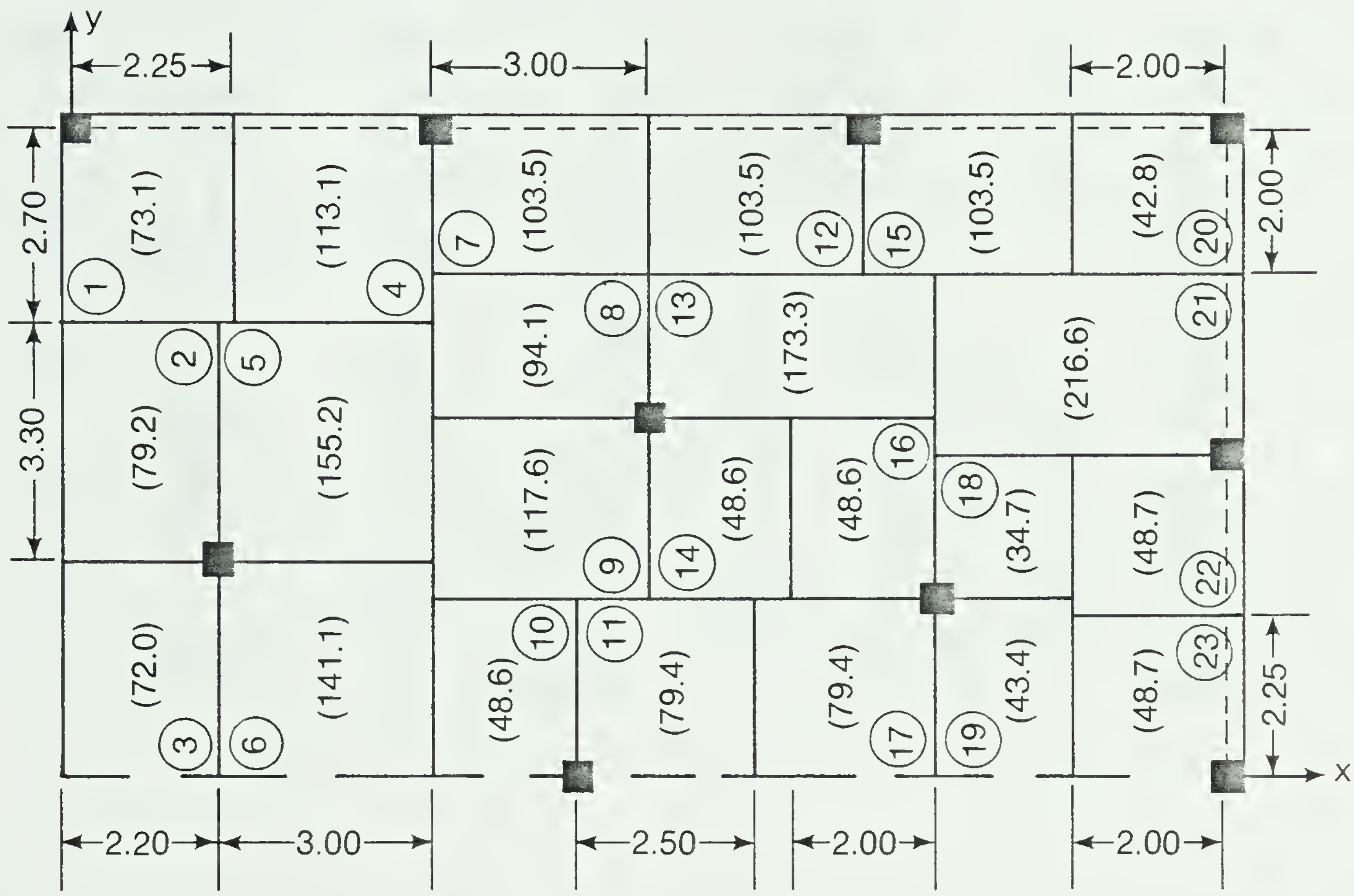


FIGURE 4.10 Initial Boundary Lines and Static Moments for Segments in the X-Direction - Example II

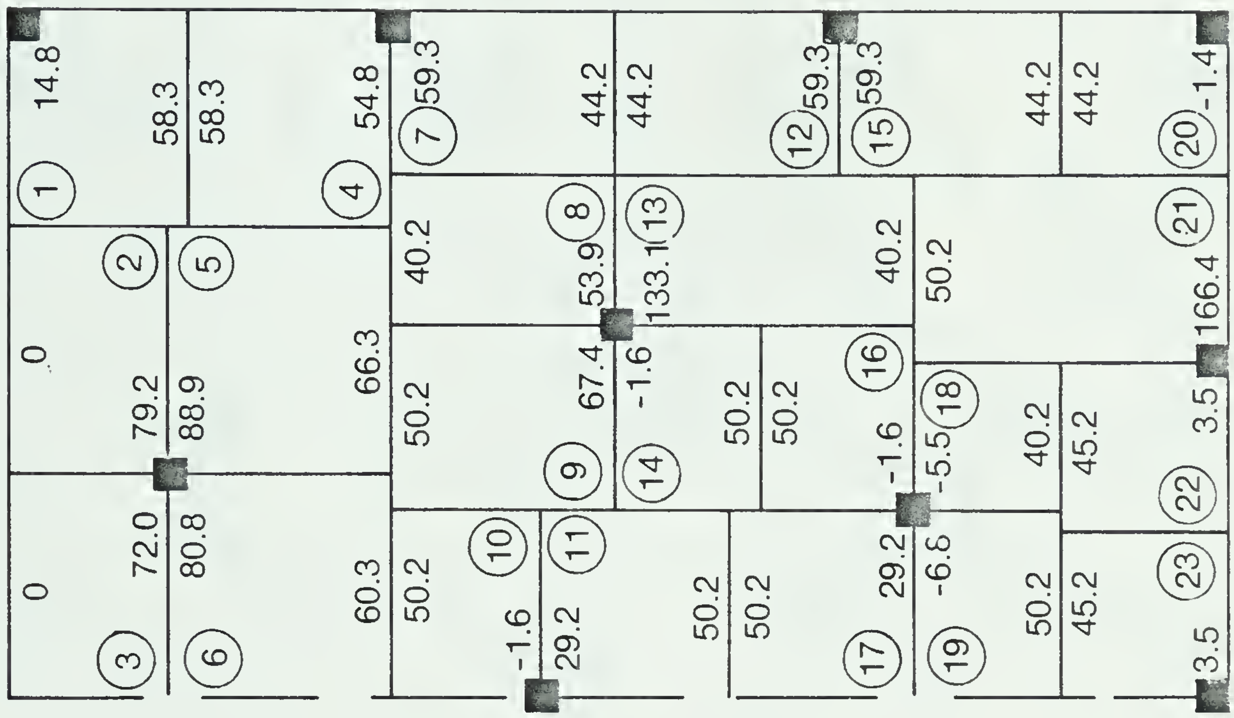


FIGURE 4.11 Initial Negative and Positive Segment Moments in the X-Direction

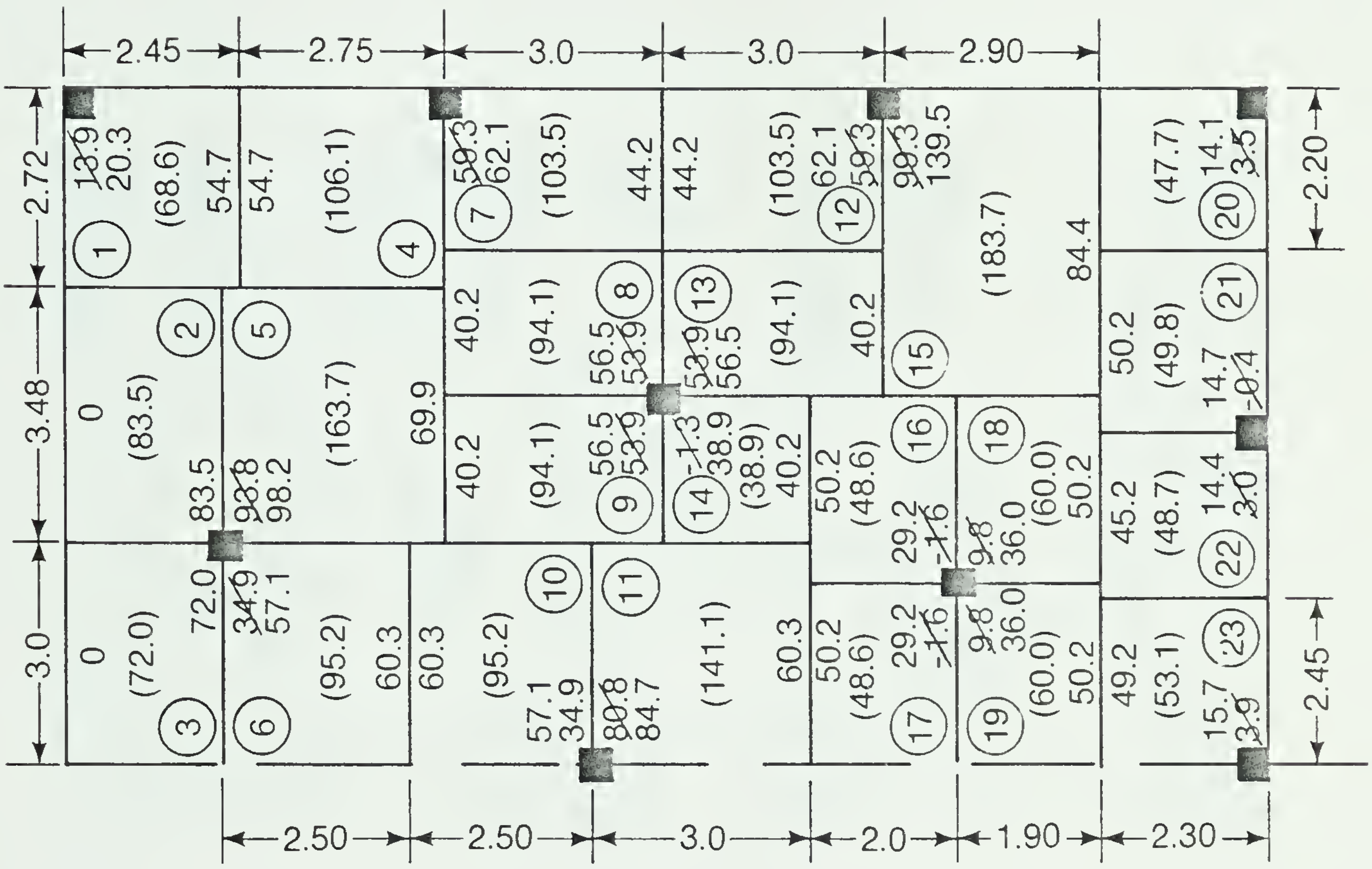


FIGURE 4.12 Modifications to X-Direction Moments from Initial Trial

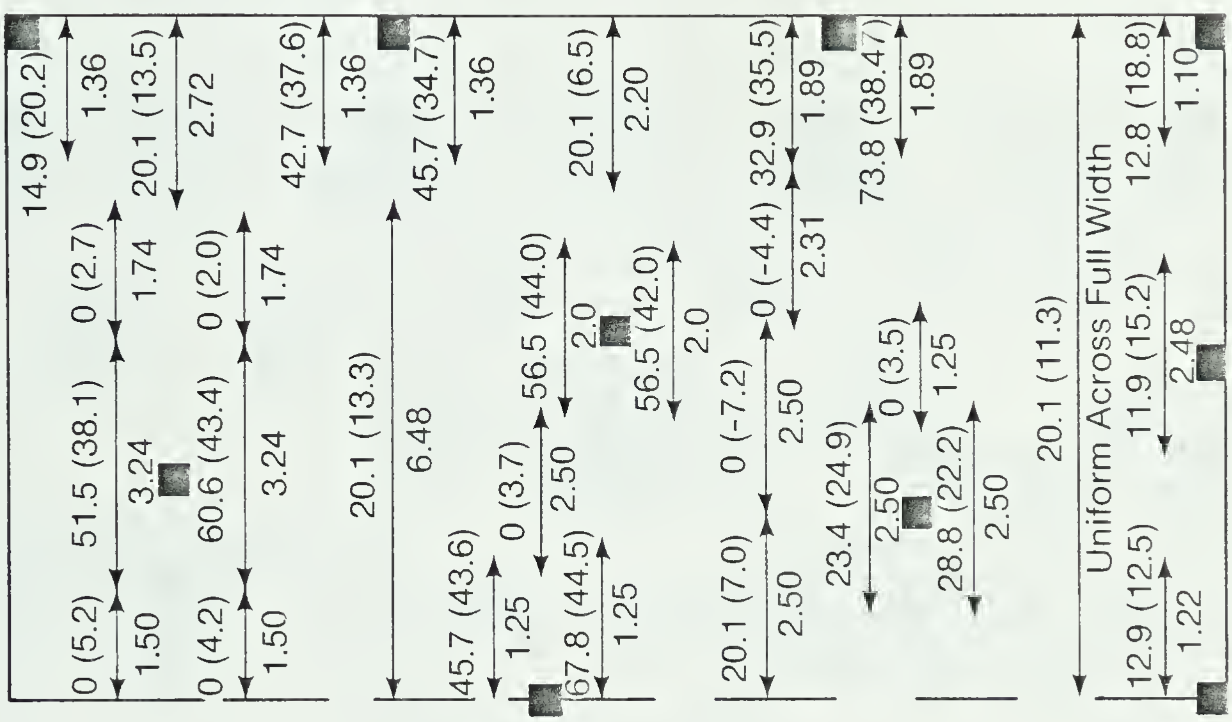


FIGURE 4.13 Distribution of X-Moments - Example II

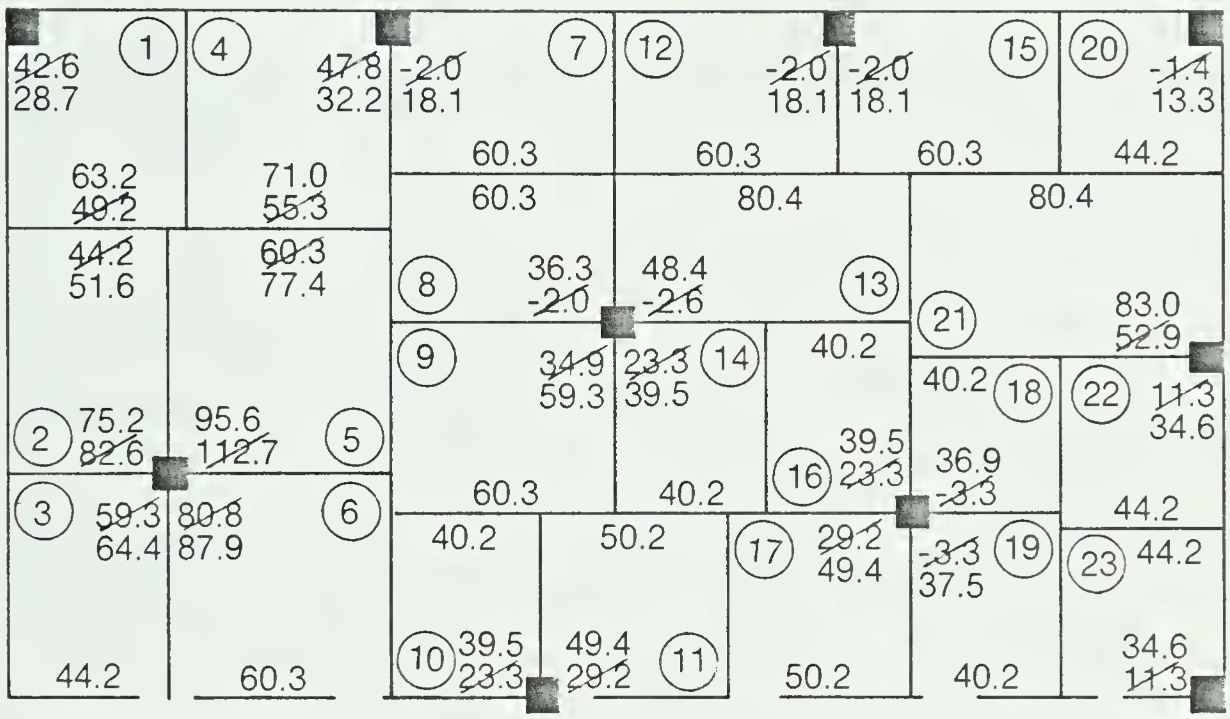


FIGURE 4.14 Negative and Positive Segment Moments in the Y-Direction

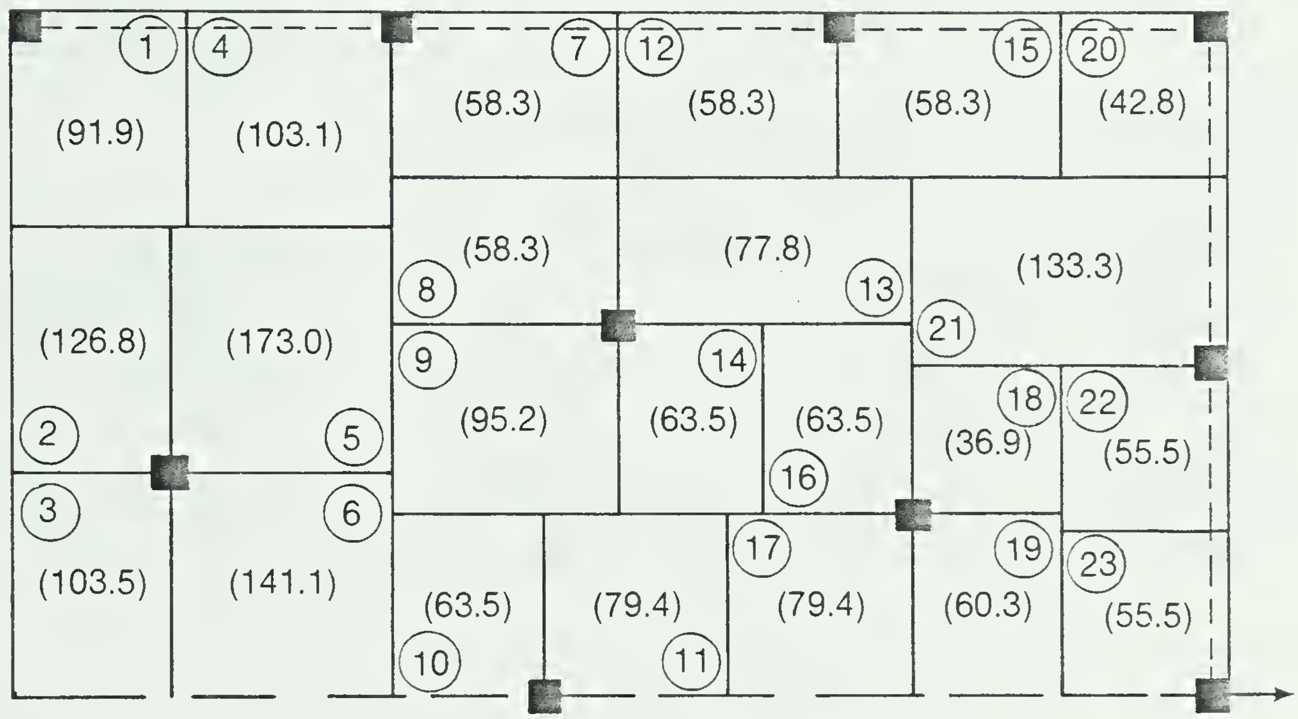


FIGURE 4.15 Initial Segment Moments in the Y-Direction - Example II

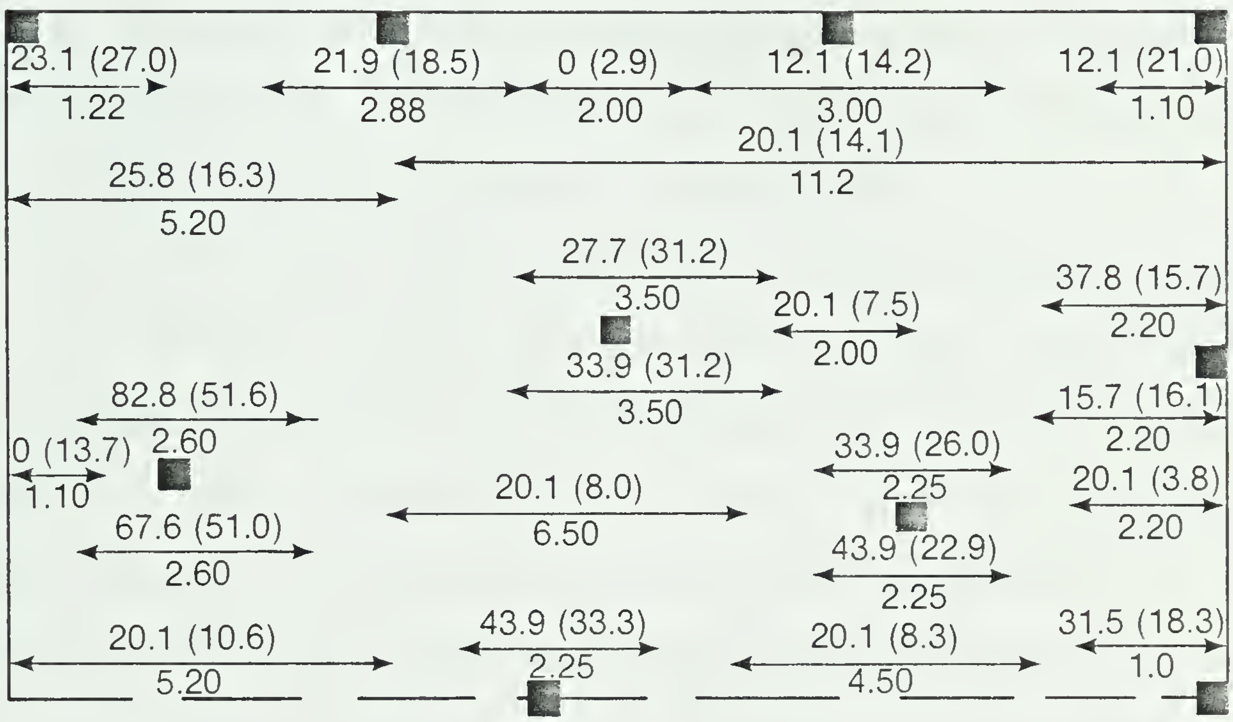


FIGURE 4.16 Distribution of Y-Moments - Example II

5. Design of Flat Slabs Supported on Walls and Columns

5.1 Introduction to Design Example III

To be able to handle the case of a slab supported by walls or walls and columns two segments were developed, the edge supported and adjacent edge supported segments. Example III is intended to demonstrate the use of these two elements and how they can be used in combination with the corner supported segment in determining a lower bound moment field for a slab supported on walls and columns.

The slab under consideration is shown in plan in Fig. 5.1. All columns are 400×400 mm² and the wall thicknesses are 400 mm. The slab is built integrally with all walls, except for the L-shaped exterior wall which represents a simple support. In determining the slab thickness a governing clear span of 5.6 m was used and based on CSA-A23.3(12) an initial trial thickness of 195 mm was selected, resulting in a uniform design load of 12.0 kN/m².

5.2 Selection of Segment Boundary Lines

The segment boundary or zero shear lines are placed, for the most part, according to those assumptions dealt with in chapter 2. Firstly, for columns along an edge the zero shear lines are assumed to occur at 0.42 of the panel length. Secondly, for edge type segments along the outside boundaries the lines are moved out to 0.5 of the panel

length. Finally, the zero shear lines between interior supports are assumed located at the panel centerline. The division of the slab into segments is shown in Fig. 5.2.

There are two major things to note about the segments in Fig. 5.2. First of all the edge supported and adjacent edge supported segments share their supports with corner supported segments. Care must be taken to design for all moments in these areas and also to ensure continuity across common boundaries. Secondly, note that the corner wall in the lower right corner of the slab has unequal leg lengths. Since the aspect ratio of an adjacent edge supported segment is fixed at 1.0 this situation may be handled by using an adjacent edge segment and an edge supported segment side-by-side.

5.3 Calculation of Total Static Segment Moments

The total static moments in the two principal directions are calculated for each individual segment using equations 2.1 through 2.5. The segment moments are shown for the x and y-directions respectively in Figs. 5.3 and 5.4.

In each case there are edge supported segments with no static segment moments given. This will apply to segments where the supported edge is parallel to the direction of moments being considered since equilibrium requires no load to be transferred by moments parallel to the support.

5.4 Splitting of Segment Moments into Positive and Negative Components

The initial splitting of the total segment moments is dependent on the moment capacity of the minimum bottom steel requirements in reference 12. Selecting a bar spacing of 250 mm and 10 M bars for the bottom mat a minimum positive moment capacity is calculated of 23.3 kN-m/m. This moment is not necessarily the minimum strength requirement but meets the minimum capacity provided by shrinkage and temperature steel. From equation 2.6 the corresponding negative moments are calculated and shown by the first numbers along the support edge in Figs. 5.5 and 5.6, for the x and y-direction moments respectively.

The selection of appropriate ψ values for corner supported segments has already been discussed in the two examples of chapter 4. In this example ψ for exterior corner supported segments is taken as 0.42 and 1.65 for interior segments.

The calculated ψ values for edge supported segments follow the same guidelines as for the corner elements. Looking first at the x-direction moments in Fig. 5.5, it is seen that the ψ for the wall segment on the left side of the slab is 0.95. To produce a more economical split of positive and negative moments this is reduced to 0.5 by increasing the positive moment to 30.0 kN-m/m. This can be accomplished by decreasing the bottom steel spacing to 160 mm in the panel bounded by support lines C and D and 1 and 2 (see Fig.

5.1). The other exterior edge segment is simply supported so there is no negative moment to develop. To maintain equilibrium of the segment the minimum positive moment capacity must be increased to 32.3 kN-m/m. The interior edge supported segment, with support parallel to the y-axis, has its negative moment adjusted to give a ψ value of 1.60.

In the y-direction there are two edge supported segments in the top panel. To maintain the minimum positive moment between them the span boundaries are shifted upwards in line with the span boundary to the left. This has the effect of decreasing the negative moment in the exterior segment and increasing the negative moment in the interior segment. The same boundary change was applied to the four corner supported segments bounded by support lines A and B and 4 and 5 (Fig. 5.1) as shown by the dashed line in Fig. 5.6.

For the slab there are two adjacent edge supported segments to consider, the exterior, simply supported segment and the fixed supported, interior segment. For the exterior one the positive span moment must be equal to the total static segment moment. This requires an increase in the span boundary moment of 22.3 kN-m. In addition, 30% of the total segment moment must be provided along the wall to prevent any adverse effects due to twisting moments at the supports. The fixed supported adjacent segment has a very low ψ value of 0.24. To resist any twisting moments which may developed the support moment is increased so ψ is equal to 1.35. The

same holds true for the y-direction moments in these segments.

All changes to the positive and negative moments in both directions are given in Figs. 5.5 and 5.6, below the initial crossed out moments.

5.5 Distribution of Edge Moments along Segment Boundaries

Having a fairly regular layout the distribution of edge moments is straightforward and follows the recommendations in chapter 3.

When the negative edge moments from corner supported segments, next to adjacent edge segments, extend across the span boundary of the adjacent element it has been found that the moment is concentrated at the wall edge. The selected values of α or ϵ for distributing this moment are taken equal to 0.4 of the adjacent edge segment span length. The top mat used to resist the moments in this region is considered separate and extra to the other steel placed within the segment boundaries. For the case where the negative moment does not extend across the adjacent segment boundaries the usual values of α and ϵ apply.

The negative moments in an edge supported segment are distributed uniformly across the support edge resulting in a uniform top mat (see Fig. 2.2a). For simply supported edge segments no top mat is necessary.

The negative moments in adjacent edge segments are distributed according to Fig. 2.2b with $M_{1-\alpha}/M_{\alpha}$ and $M_{1-\varepsilon}/M_{\varepsilon}$ equal to 2.0. Where the support edge is simply supported the extra negative moment due to twisting is distributed with the M^- ratios inverted (see Fig. 3.12).

The distribution of edge moments is shown in Figs. 5.7 and 5.8 for the x and y-direction moments respectively. The number above the line represents the moment in kN-m/m and the number below the line is the distance over which the moment is to be distributed.

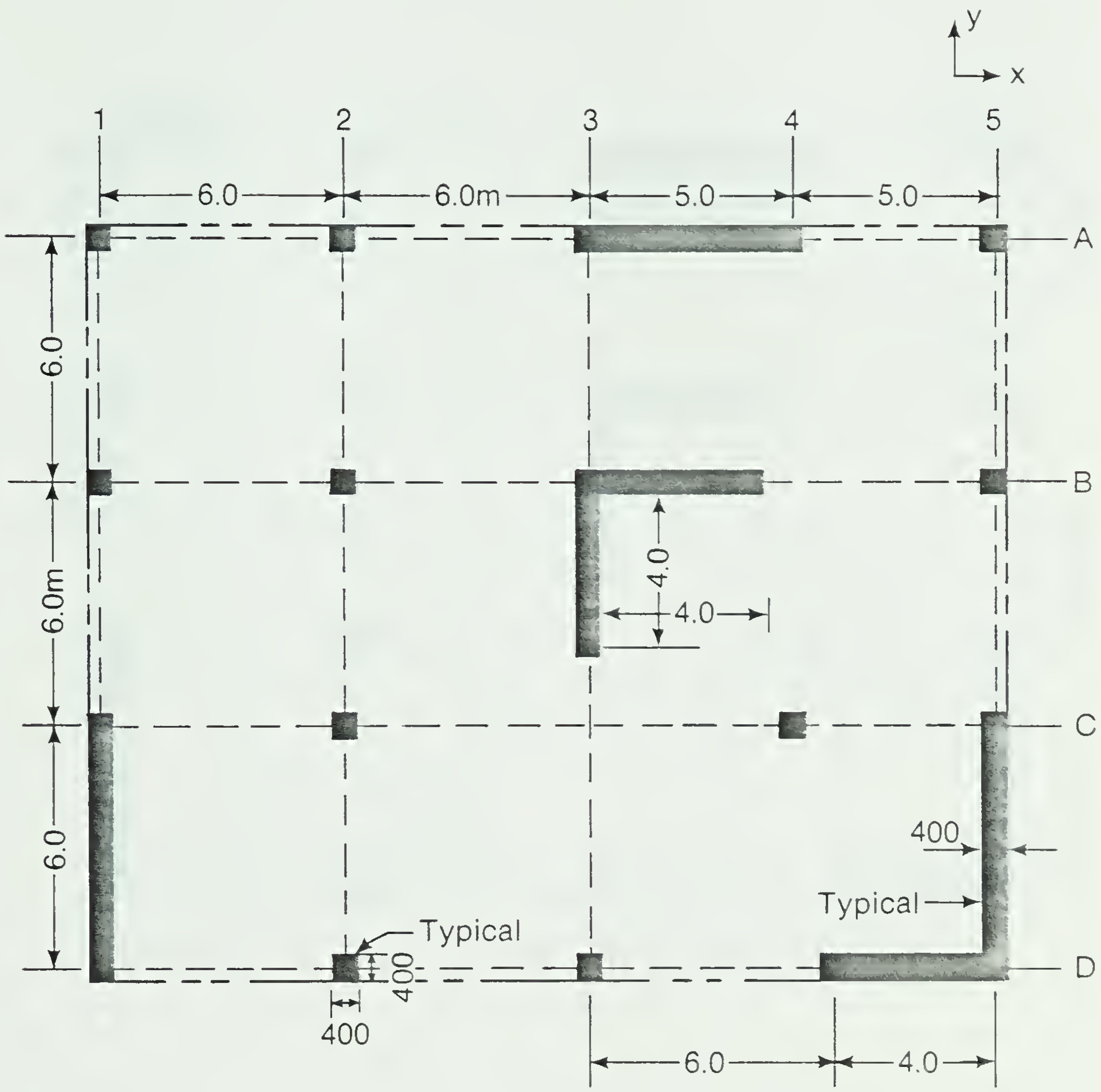


FIGURE 5.1 Plan View - Example III

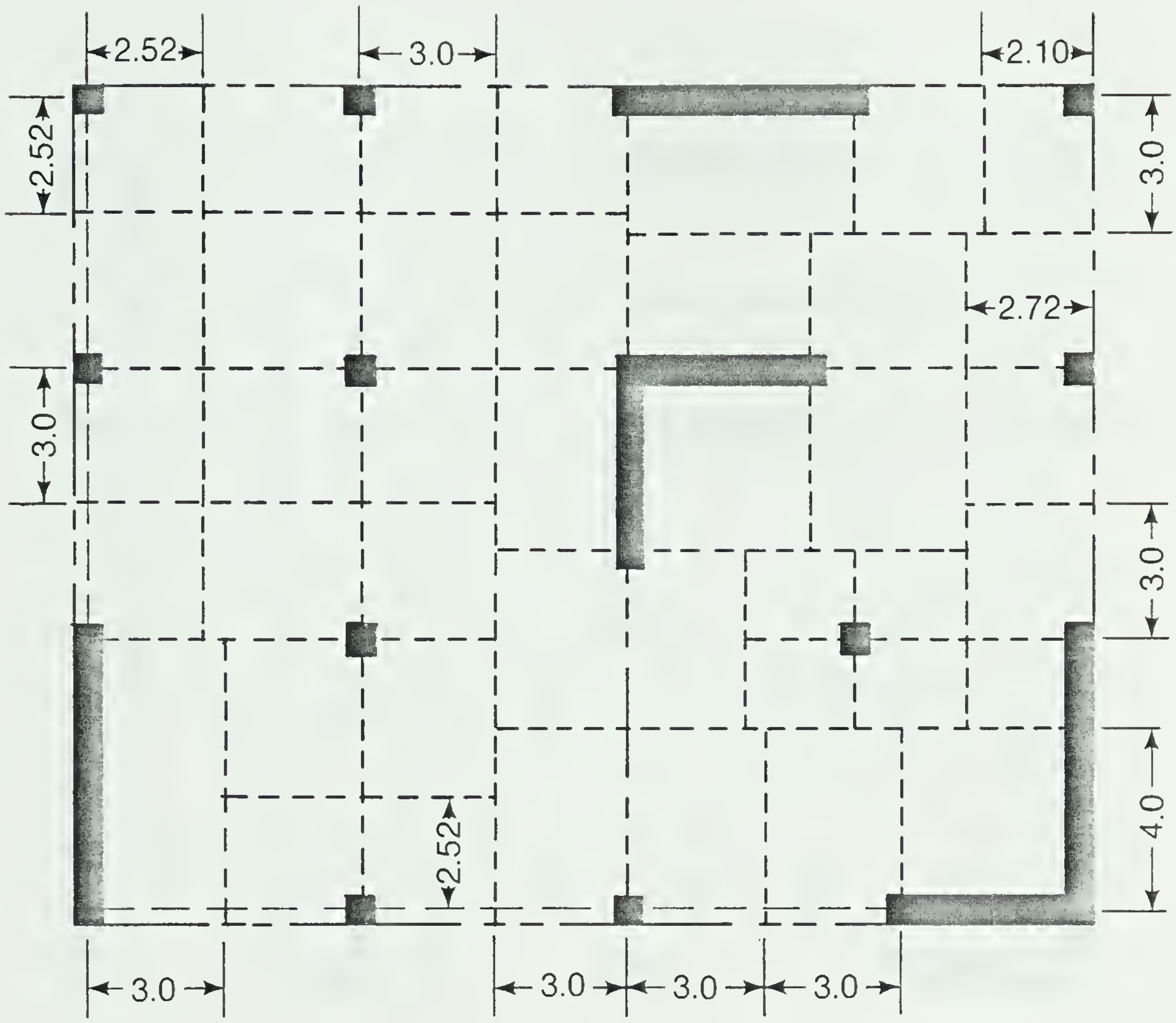


FIGURE 5.2 Initial Segment Boundary Lines for Example III

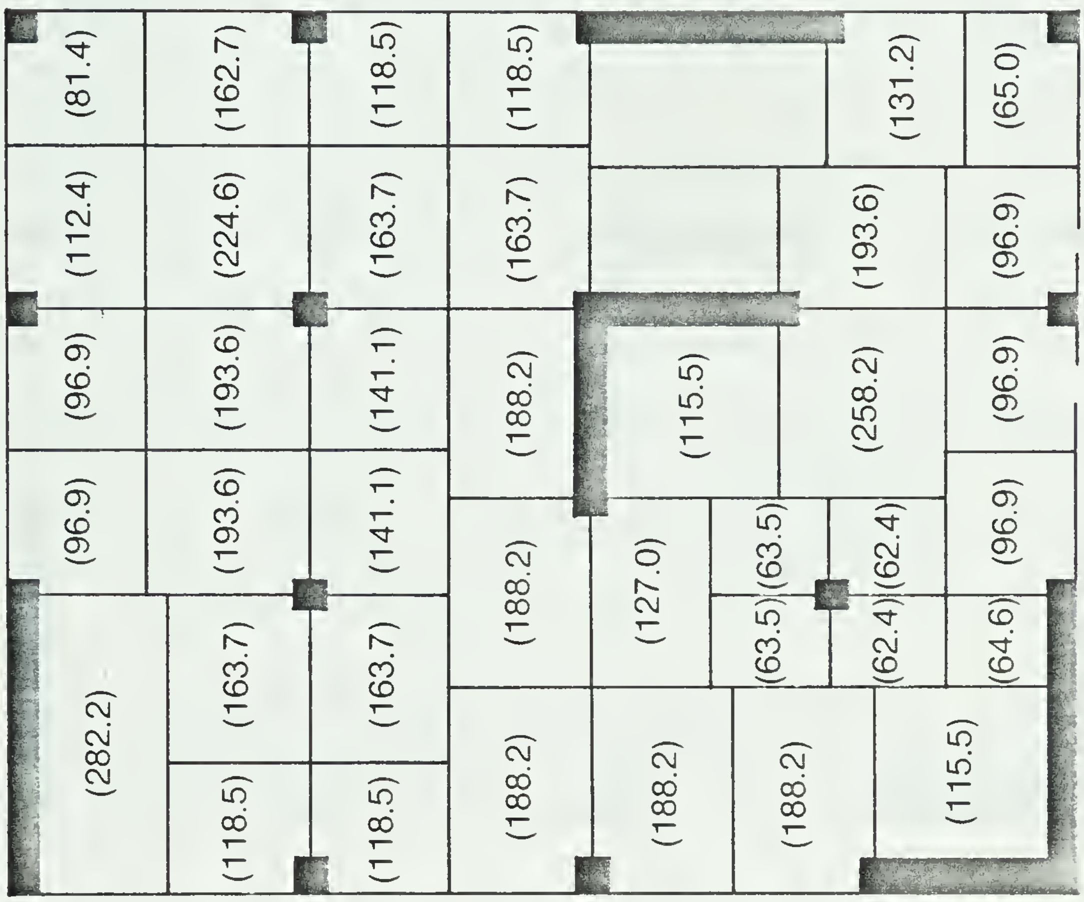


FIGURE 5.3 Total Static Segment Moments in the X-Direction for Example III

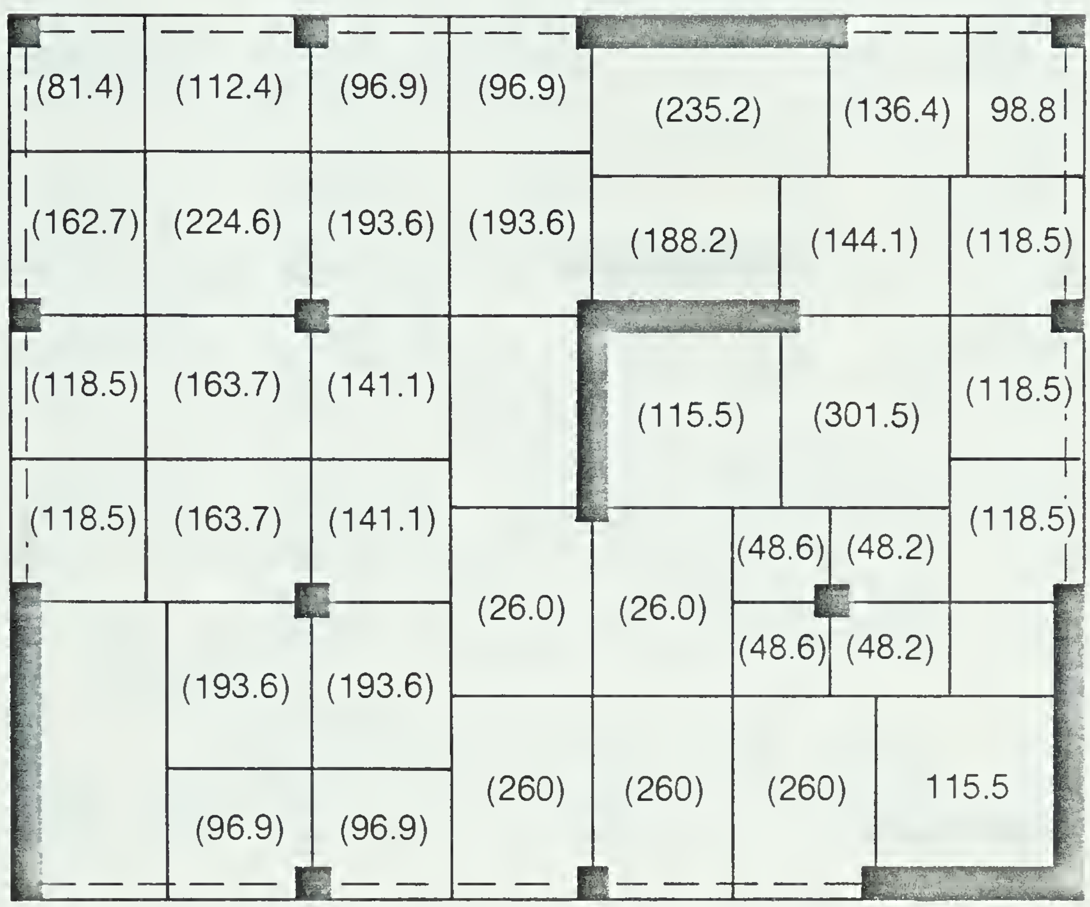


FIGURE 5.4 Total Static Segment Moments in the Y-Direction for Example III

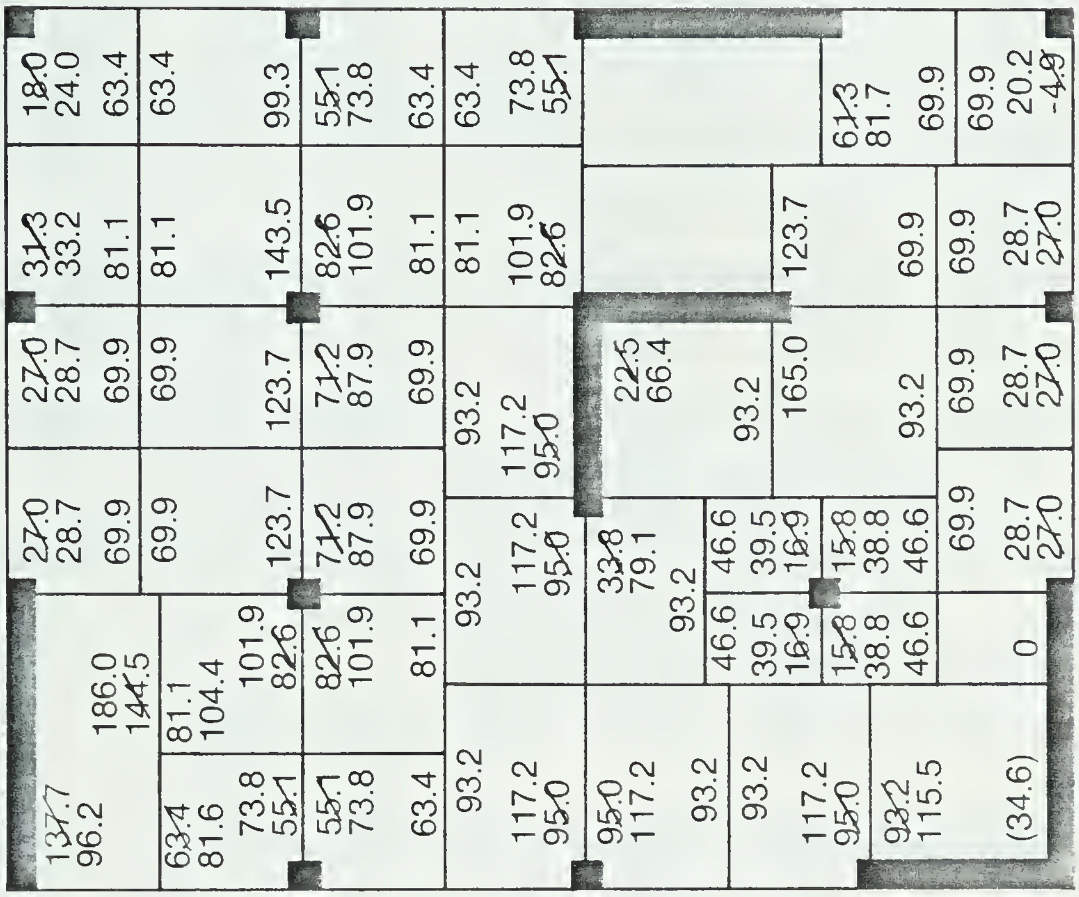


FIGURE 5.5 Positive and Negative Moments in the X-Direction for Example III

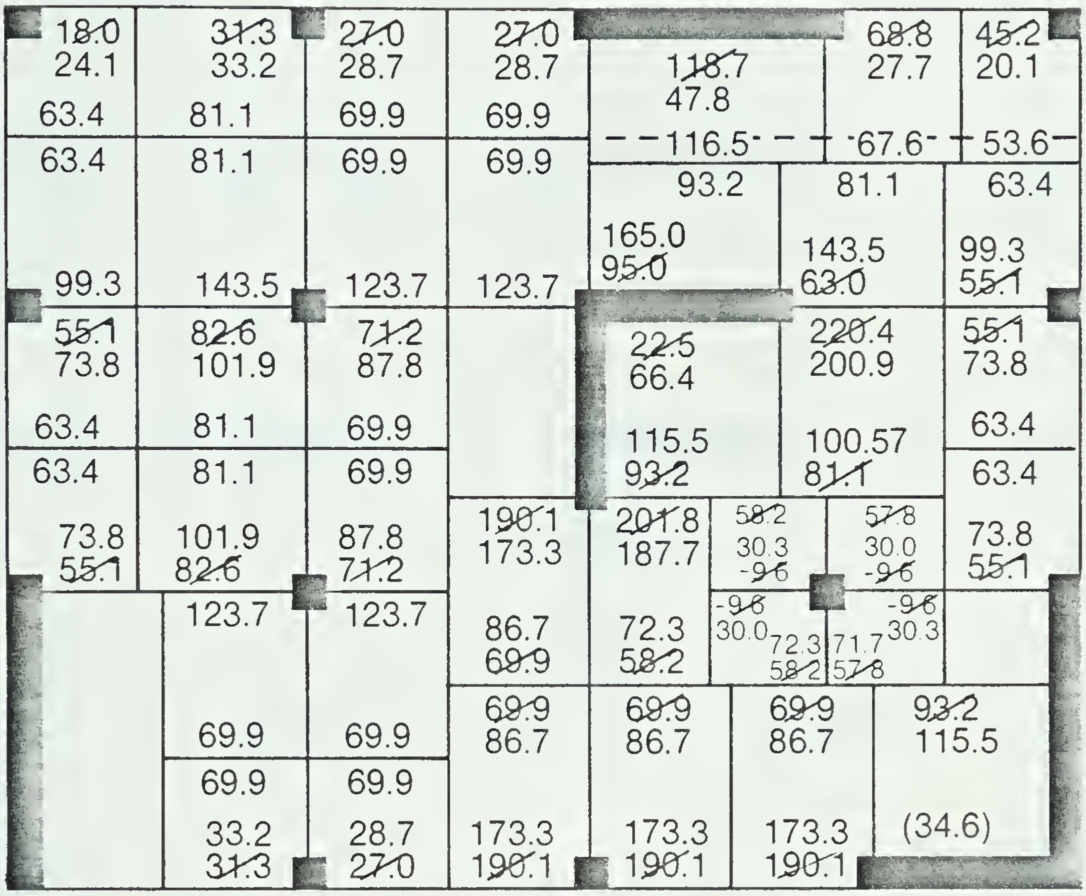


FIGURE 5.6 Positive and Negative Moments in the Y-Direction for Example III

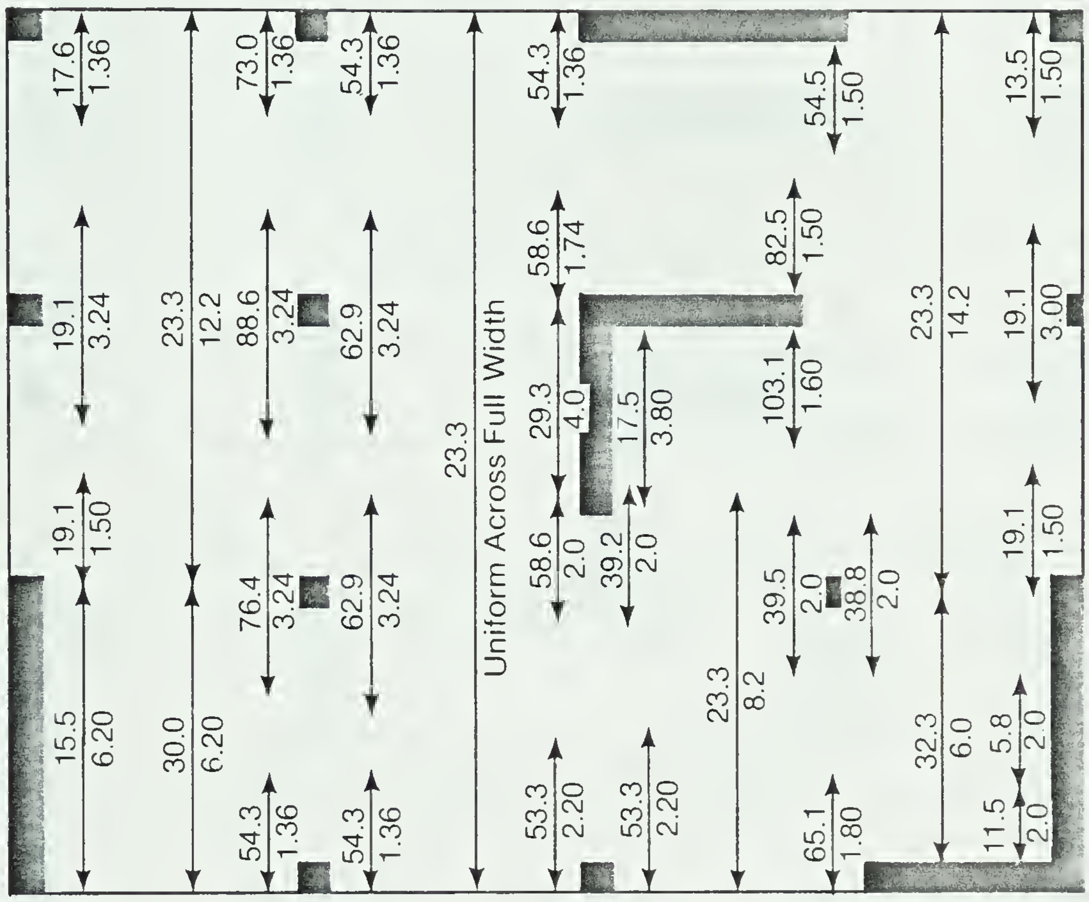


FIGURE 5.7 Distribution of X-Moments - Example III

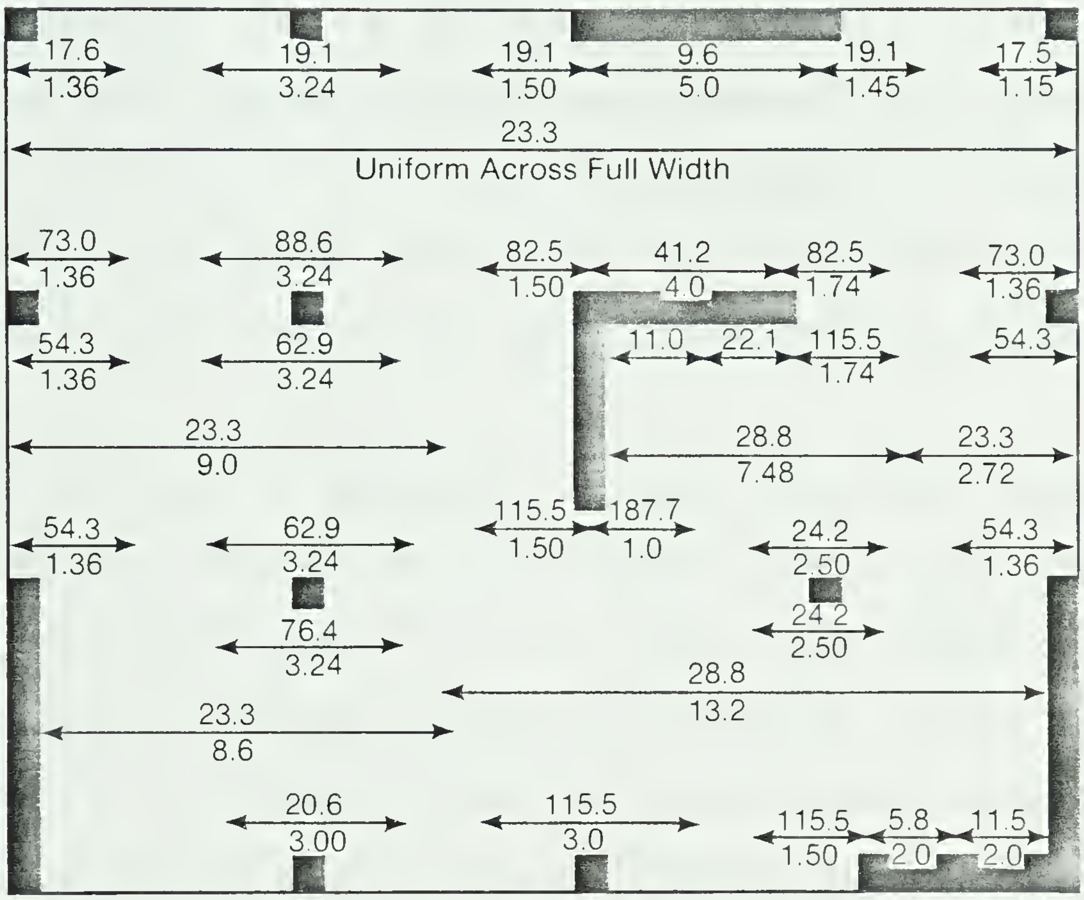


FIGURE 5.8 Distribution of Y-Moments - Example III

6. Summary and Conclusions

6.1 Summary

The purpose of this thesis was to develop a lower bound design procedure for reinforced concrete flat slabs that is both simple in approach and easy to apply. In the Segment Design Method a slab is divided into a number of rectangular segments, each segment bounded by lines of zero shear. Boundary moments are assigned to each segment to satisfy equilibrium and moment continuity at the boundaries. The segments together provide a complete moment field for the slab.

Three types of segments, a corner supported segment, an edge supported segment and an adjacent edge supported segment were developed for use. These three segments allow the designer to handle regular or irregular slabs, supported on columns, or columns and walls. Each segment type was analysed using an elastic finite element program. From these analyses, simplified moment fields were developed which resulted in satisfactary boundary moment distributions and aided in selecting theoretical cutoff points for the reinforcing.

The philosophy behind the design method has developed from modifications and extensions to existing equilibrium techniques.

6.2 Conclusions

The Segment Design Method is a viable procedure for the design of regular and irregular, reinforced concrete flat slabs. The steps in the methodology are straightforward without tedious calculations and modifications to the design moments, if necessary, involve little effort. By using a moment distribution which gives uniform positive moments over the slab and concentrated negative moments over the supports the required reinforcing layout is kept simple. The results are lower bound thereby always providing a slab on the safe side.

The SDM has the further advantage that the designer has the option of obtaining several reinforcing layouts and selecting the best one as his final design.

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Appendix A

Design Aids for Selection of Reinforcing Steel

The two tables presented in this appendix are to serve as a guide in selectiong reinforcement for both the top and bottom mats. Moment capacities are given, for constant material properties and bar size, for various slab thicknesses and bar spacings.

The constants used in each of the tables are given below.

Strength of concrete..........................30 MPa

Yield strength of reinforcement..............400 MPa

Table A.1 gives the moment capacities for 10 M bars and Table A.2 gives them for 15 M bars. The slab thicknesses vary from 100 to 220 mm.

Equation A.1 is used to calculate the moment capacities and is based on bar areas of 100 mm^2 for No. 10 M bars and 200 mm^2 for No. 15 M bars.

$$Mu=\phi As\times fy[d-(As\times fy/1700f'c)] \qquad A.1$$

By entering the appropriate table, depending on which size bars are used, with the desired spacing and slab thickness the designer can find the corresponding moment capacity.

Linear interpolation is to used for inbetween values.

Table A.1 Moment Capacities (kN-m/m) of 10 M Bars at Various Spacings and Slab Thicknesses with 20 mm Cover

Bar Space mm	Slab Thickness h (mm)								
	100	125	150	170	180	190	200	210	220
50	39.1	57.1	75.1	89.5	96.7	103.9	111.1	118.3	125.5
75	28.6	45.4	52.6	62.2	67.0	71.8	76.6	81.4	86.2
100	22.4	31.4	40.4	47.6	51.2	54.8	58.4	62.0	65.6
125	18.4	25.6	32.8	38.5	41.4	44.3	47.2	50.0	52.9
150	15.5	23.9	27.5	32.3	34.7	37.1	39.5	41.9	44.3
175	13.5	20.7	23.8	27.9	29.9	32.0	34.0	36.1	38.2
200	11.9	16.4	20.9	24.5	26.3	28.1	29.9	31.7	33.5
225		14.6	18.6	21.8	23.4	25.0	26.6	28.2	29.8
250		13.2	16.8	19.7	21.1	22.6	24.0	25.5	26.9
275			15.3	18.0	19.3	20.6	21.9	23.2	24.5
300			14.1	16.5	17.7	18.9	20.1	21.3	22.5
340				14.6	15.6	16.7	17.8	18.8	19.8
360					14.8	15.8	16.8	17.8	18.8
380						15.0	15.9	16.9	17.8
400							15.1	16.0	16.9
420								15.3	16.1
440									15.4

Table A.2 Moment Capacities (kN-m/m) of 15 M Bars at Various Spacings and Slab Thicknesses with 20 mm Cover

Bar Space mm	Slab Thickness h (mm)								
	100	125	150	170	180	190	200	210	220
50	47.0	83.0	119.0	147.8	162.2	276.6	191.0	205.4	219.8
75	41.4	65.4	89.4	108.6	118.2	127.8	137.4	147.0	156.6
100	34.8	52.8	70.8	85.2	92.4	99.6	106.8	114.0	121.2
125	29.6	44.0	58.4	70.0	75.7	81.5	87.2	93.0	98.8
150	25.7	37.7	49.7	59.3	64.1	68.9	73.7	78.5	83.3
175	22.6	32.9	43.2	51.4	55.6	59.7	63.8	67.9	72.0
200	20.2	29.2	38.2	45.4	49.0	52.6	56.2	59.8	63.4
225		26.2	34.2	40.6	43.8	47.0	50.2	53.4	56.6
250		23.8	31.0	36.8	39.7	42.5	45.4	48.3	51.2
275			28.4	33.6	36.2	38.8	41.4	44.1	46.7
300			26.1	30.9	33.3	35.7	38.1	40.5	42.9
340				27.4	29.5	31.6	33.8	35.9	38.0
360					27.9	29.9	31.9	33.9	35.9
380						28.4	30.3	32.2	34.1
400							28.8	30.6	32.4
420								27.9	30.9
440									29.5